ADAPTIVE BLIND IDENTIFICATION OF SIMO SYSTEMS USING CHANNEL CROSS-RELATION IN THE FREQUENCY DOMAIN

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ABSTRACT

The implementation of existing methods for blind identification of single-input multiple-output (SIMO) systems is limited in practice since they are difficult to execute in an adaptive mode and are in general computationally intensive. We extend our previous study into the frequency domain and propose an unconstrained normalized multi-channel frequency-domain LMS (UNMCFLMS) algorithm. Numerical simulations show that the UNMCFLMS algorithm performs as well as (for a SIMO system with relatively short channel impulse responses) or better than (for a SIMO system with long channel impulse responses) its time-domain counterpart and the cross-relation (CR) batch method in practical situations.

1. INTRODUCTION

Blind identification of single-input multiple-output (SIMO) systems has attracted considerable attention recently because of its extensive applications in signal processing and communications. Approaches based on second-order statistics (SOS) of system's outputs [1], [2], [3], [4], [5] are deemed more attractive, because of their fast convergence, than higher-order statistics (HOS) methods [6]. However, existing SOS methods are not satisfactory because they are difficult to implement in an adaptive mode and are in general computationally intensive [7].

In an earlier study [8], we found a systematic way to construct an error signal exploiting the cross relations between different channels and proposed two time-domain adaptive algorithms. It was shown that they converge in the mean to the real channel impulse responses. But they are either slow in convergence or complicated in computation. In this paper, we continue our exploration of this problem in the frequency domain and try to develop an improved efficient adaptive filter. An unconstrained normalized multi-channel frequency-domain LMS (UNMCFLMS) algorithm is proposed and experimental results show some promise for its success.

2. SIGNAL MODEL AND PROBLEM FORMULATION

In an FIR SIMO linear system, the i-th channel output signal $x_i(n)$ is the result of a linear convolution between the source signal $s(n)$ and the corresponding true (subscript t) channel impulse response $h_{t,i}$, corrupted by an additive background noise $b_i(n)$:

$$x_i(n) = h_{t,i} * s(n) + b_i(n), \;\; i = 1, 2, ..., M, \qquad (1)$$

where $*$ stands for linear convolution and M is the number of channels. In vector form, (1) can be expressed as:

$$\mathbf{x}_i(n) = \mathbf{H}_{t,i} \cdot \mathbf{s}(n) + \mathbf{b}_i(n), \qquad (2)$$

where

$$\begin{aligned}
\mathbf{x}_i(n) &= [x_i(n)\;\, x_i(n-1)\;\, \cdots\;\, x_i(n-L+1)]^T, \\
\mathbf{H}_{t,i} &= \begin{bmatrix} h_{t,i,0} & \cdots & h_{t,i,L-1} & \cdots & 0 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 0 & \cdots & h_{t,i,0} & \cdots & h_{t,i,L-1} \end{bmatrix}, \\
\mathbf{s}(n) &= [s(n)\;\, s(n-1)\;\, \cdots\;\, s(n-2L+2)]^T, \\
\mathbf{b}_i(n) &= [b_i(n)\;\, b_i(n-1)\;\, \cdots\;\, b_i(n-L+1)]^T,
\end{aligned}$$

L is set to the length of the longest channel impulse response by assumption, and $(\cdot)^T$ denotes vector/matrix transpose. The channel parameter matrix $\mathbf{H}_{t,i}$ is of dimension $L \times (2L-1)$ and is constructed from the channel's impulse response:

$$\mathbf{h}_{t,i} = [h_{t,i,0}\;\, h_{t,i,1}\;\, \cdots\;\, h_{t,i,L-1}]^T. \qquad (3)$$

Moreover, the additive noise components in different channels are assumed to be uncorrelated with the source signal even though they might be mutually dependent.

A blind system identification algorithm is to estimate the channel impulse responses $\mathbf{h}_i$ ($i = 1, 2, ..., M$) from the observations $\mathbf{x}_i$ without utilizing the source signal $\mathbf{s}(n)$. The following two assumptions are made throughout this paper to guarantee an identifiable system using only the second-order statistics [4]:

1. The polynomials formed from $\mathbf{h}_{t,i}, i = 1, 2, ...M$, are co-prime, i.e., the channel transfer functions $H_{t,i}(z)$ do not share any common zeros;
2. The autocorrelation matrix $\mathbf{R}_{ss} = E\left\{\mathbf{s}(n)\mathbf{s}^T(n)\right\}$ of the source signal is of full rank.

3. THE PRINCIPLE OF ADAPTIVE BLIND SYSTEM IDENTIFICATION

Basically, a multi-channel system can be blindly identified because of the channel diversity which makes the outputs of different channels distinct though related. By following the fact that

$$x_i(n) * h_{t,j} = s(n) * h_{t,i} * h_{t,j} = x_j(n) * h_{t,i}, \qquad (4)$$

a cross-relation between the i-th and j-th channel outputs, in the absence of noise, can be formulated as

$$\mathbf{x}_i^T(n)\mathbf{h}_{t,j} = \mathbf{x}_j^T(n)\mathbf{h}_{t,i}, \;\; i, j = 1, 2, ..., M, \; i \neq j. \qquad (5)$$

When noise is present or the channel impulse responses are improperly modeled, the left and right hand sides of (5) are generally not equal and the inequality can be used to define an *a priori* error signal as follows [8]:

$$e_{ij}(n+1) = \frac{\mathbf{x}_i^T(n+1)\mathbf{h}_j(n) - \mathbf{x}_j^T(n+1)\mathbf{h}_i(n)}{\|\mathbf{h}(n)\|}, \quad (6)$$

where $\mathbf{h}_i(n)$ is the model filter for the i-th channel at time n and

$$\mathbf{h}(n) = \left[\begin{array}{ccc} \mathbf{h}_1^T(n) & \mathbf{h}_2^T(n) & \cdots \mathbf{h}_M^T(n) \end{array} \right]^T.$$

The model filter is normalized in order to avoid a trivial solution whose elements are all zeros. Based on the error signal defined here, a cost function at time $n+1$ is given by

$$J(n+1) = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} e_{ij}^2(n+1). \quad (7)$$

An adaptive algorithm is then derived to efficiently determine the model filter $\mathbf{h}_i$ that minimizes this cost function and therefore would be a good estimate of $\mathbf{h}_{t,i}/\|\mathbf{h}_t\|$ $(i = 1, 2, ..., M)$.

4. A FREQUENCY-DOMAIN ADAPTIVE ALGORITHM

The time-domain adaptive algorithms proposed in [8] are either slow in convergence (the multi-channel LMS algorithm) or inefficient (the multi-channel Newton algorithm). Here, we will develop an adaptive blind channel identification algorithm in the frequency domain to seek for a good balance between fast convergence and low computational complexity. In the following derivation, matrices and vectors in the frequency domain are represented respectively by uppercase calligraphic and lowercase bold italic letters, and a vector is further emphasized by an underbar.

To begin, we define an intermediate signal $y_{ij} \triangleq x_i * h_j$, the convolution result of the i-th channel output x_i and the j-th model filter h_j. In vector form, a block of such a signal can be expressed in the frequency domain as

$$\underline{\boldsymbol{y}}_{ij}(m+1) = \mathcal{W}_{L\times 2L}^{01}\mathcal{D}_{x_i}(m+1)\mathcal{W}_{2L\times L}^{10}\underline{\boldsymbol{h}}_j(m), \quad (8)$$

where

$$\begin{aligned}
\mathcal{W}_{L\times 2L}^{01} &= \mathbf{F}_{L\times L}\left[\begin{array}{cc} \mathbf{0}_{L\times L} & \mathbf{I}_{L\times L} \end{array} \right]\mathbf{F}_{2L\times 2L}^{-1}, \\
\mathcal{D}_{x_i}(m+1) &= \mathrm{diag}\left\{\mathbf{F}_{2L\times 2L}\cdot \mathbf{x}_i(m+1)_{2L\times 1}\right\}, \\
\mathcal{W}_{2L\times L}^{10} &= \mathbf{F}_{2L\times 2L}\left[\begin{array}{cc} \mathbf{I}_{L\times L} & \mathbf{0}_{L\times L} \end{array} \right]^T\mathbf{F}_{L\times L}^{-1}, \\
\underline{\boldsymbol{h}}_j(m) &= \mathbf{F}_{L\times L}\mathbf{h}_j(m), \\
\mathbf{x}_i(m+1)_{2L\times 1} &= \left[\begin{array}{ccc} x_i(mL) & x_i(mL+1) & \cdots \end{array} \right. \\
&\qquad \left. x_i(mL+2L-1)\right]^T, \quad (9)
\end{aligned}$$

$\mathbf{F}_{L\times L}$ and $\mathbf{F}_{L\times L}^{-1}$ are respectively the Fourier and inverse Fourier matrices of size $L \times L$, and m is the block time index. Then a block of the error signal based on the cross-relation between the i-th and the j-th channel in the frequency domain is determined as:

$$\begin{aligned}
\underline{\boldsymbol{e}}_{ij}(m+1) &= \underline{\boldsymbol{y}}_{ij}(m+1) - \underline{\boldsymbol{y}}_{ji}(m+1) \\
&= \mathcal{W}_{L\times 2L}^{01}\left[\mathcal{D}_{x_i}(m+1)\mathcal{W}_{2L\times L}^{10}\underline{\boldsymbol{h}}_j(m) - \right. \\
&\qquad \left. \mathcal{D}_{x_j}(m+1)\mathcal{W}_{2L\times L}^{10}\underline{\boldsymbol{h}}_i(m)\right]. \quad (10)
\end{aligned}$$

Continuing, we construct a (frequency-domain) cost function at the $(m+1)$-th block as follows:

$$J_f(m+1) = \sum_{i=1}^{M-1} \sum_{j=i+1}^{M} \underline{\boldsymbol{e}}_{ij}^H(m+1)\underline{\boldsymbol{e}}_{ij}(m+1), \quad (11)$$

where $(\cdot)^H$ denotes Hermitian transpose. Therefore, by minimizing the mean value of $J_f(m+1)$, the model filter in the frequency domain can be updated as:

$$\underline{\boldsymbol{h}}_k(m+1) = \underline{\boldsymbol{h}}_k(m) - \mu_f \frac{\partial J_f(m+1)}{\partial \underline{\boldsymbol{h}}_k^*(m)}, \; k = 1, 2, ..., M, \; (12)$$

where $(\cdot)^*$ stands for complex conjugate and μ_f is a small positive step size. It can be shown that

$$\frac{\partial J_f(m+1)}{\partial \underline{\boldsymbol{h}}_k^*(m)} = \sum_{i=1}^{M} \left[\mathcal{W}_{L\times 2L}^{01}\mathcal{D}_{x_i}(m+1)\mathcal{W}_{2L\times L}^{10}\right]^H \underline{\boldsymbol{e}}_{ik}(m+1), \; (13)$$

Substituting (13) into (12) yields a multi-channel frequency-domain LMS (MCFLMS) algorithm:

$$\underline{\boldsymbol{h}}_k(m+1) = \underline{\boldsymbol{h}}_k(m) - \mu_f\mathcal{W}_{L\times 2L}^{10}\sum_{i=1}^{M}\mathcal{D}_{x_i}^*(m+1)\mathcal{W}_{2L\times L}^{01}\underline{\boldsymbol{e}}_{ik}(m+1), \; (14)$$

where

$$\begin{aligned}
\mathcal{W}_{L\times 2L}^{10} &= \mathbf{F}_{L\times L}\left[\begin{array}{cc} \mathbf{I}_{L\times L} & \mathbf{0}_{L\times L} \end{array} \right]\mathbf{F}_{2L\times 2L}^{-1}, \\
\mathcal{W}_{2L\times L}^{01} &= \mathbf{F}_{2L\times 2L}\left[\begin{array}{cc} \mathbf{0}_{L\times L} & \mathbf{I}_{L\times L} \end{array} \right]^T\mathbf{F}_{L\times L}^{-1}.
\end{aligned}$$

The constraint ensuring that the adaptive algorithm would not converge to a trivial solution with all zero elements will be applied after every step of updation.

The MCFLMS is computationally more efficient compared to a multi-channel time-domain block LMS algorithm. However, the convergence of the MCFLMS algorithm is still slow because of nonuniform convergence rates of the filter coefficients and cross-coupling between them. To accelerate convergence, we will use Newton's method to develop a normalized MCFLMS (NMCFLMS) method.

By using Newton's method, we update the model filter coefficients according to:

$$\underline{\boldsymbol{h}}_k(m+1) = \underline{\boldsymbol{h}}_k(m) - \mu_f\left\{\frac{\partial}{\partial \underline{\boldsymbol{h}}_k^T(m)}\left[\frac{\partial J_f(m+1)}{\partial \underline{\boldsymbol{h}}_k^*(m)}\right]\right\}^{-1}\frac{\partial J_f(m+1)}{\partial \underline{\boldsymbol{h}}_k^*(m)}, \quad (15)$$

where the Hessian matrix can be evaluated as

$$\frac{\partial}{\partial \underline{\boldsymbol{h}}_k^T(m)}\left[\frac{\partial J_f(m+1)}{\partial \underline{\boldsymbol{h}}_k^*(m)}\right] = \mathcal{W}_{L\times 2L}^{10}\cdot \sum_{i=1, i\neq k}^{M}\left[\mathcal{D}_{x_i}^*(m+1)\mathcal{W}_{2L\times 2L}^{01}\mathcal{D}_{x_i}(m+1)\right]\mathcal{W}_{2L\times L}^{10}, \; (16)$$

and

$$\begin{aligned}\mathcal{W}^{01}_{2L\times 2L} &\triangleq \mathcal{W}^{01}_{2L\times L}\mathcal{W}^{01}_{L\times 2L} \\ &= \mathbf{F}_{2L\times 2L}\begin{bmatrix} \mathbf{0}_{L\times L} & \mathbf{0}_{L\times L} \\ \mathbf{0}_{L\times L} & \mathbf{I}_{L\times L}\end{bmatrix}\mathbf{F}^{-1}_{2L\times 2L},\end{aligned}$$

whose elements on its main diagonal dominate as shown in [9]. When L is large, $2\mathcal{W}^{01}_{2L\times 2L}$ can be well approximated by the identity matrix

$$2\mathcal{W}^{01}_{2L\times 2L} \approx \mathbf{I}_{2L\times 2L}. \quad (17)$$

Thereafter, Eq. (16) becomes

$$\frac{\partial}{\partial \underline{\boldsymbol{h}}_k^T(m)}\left[\frac{\partial J_f(m+1)}{\partial \underline{\boldsymbol{h}}_k^*(m)}\right] \approx \frac{1}{2}\mathcal{W}^{10}_{L\times 2L}\mathcal{P}_k(m+1)\mathcal{W}^{10}_{2L\times L}, \quad (18)$$

where

$$\mathcal{P}_k(m+1) = \sum_{i=1,i\neq k}^{M} \mathcal{D}^*_{x_i}(m+1)\mathcal{D}_{x_i}(m+1),\ \ k=1,2,...,M.$$

Substituting (13) and (18) into (15) and multiplying by $\mathcal{W}^{10}_{2L\times L}$ produces the *constrained* NMCFLMS algorithm:

$$\begin{aligned}&\underline{\boldsymbol{h}}_k^{10}(m+1) = \underline{\boldsymbol{h}}_k^{10}(m) - \\ &\quad 2\mu_f\mathcal{W}^{10}_{2L\times 2L}\mathcal{P}_k^{-1}(m+1)\sum_{i=1}^{M}\mathcal{D}^*_{x_i}(m+1)\underline{\boldsymbol{e}}^{01}_{ik}(m+1), \quad (19)\end{aligned}$$

where

$$\begin{aligned}\underline{\boldsymbol{h}}_k^{10}(m) &= \mathcal{W}^{10}_{2L\times L}\underline{\boldsymbol{h}}_k(m), \\ \underline{\boldsymbol{e}}^{01}_{ik}(m+1) &= \mathcal{W}^{01}_{2L\times L}\underline{\boldsymbol{e}}_{ik}(m+1), \\ \mathcal{W}^{10}_{2L\times 2L} &= \mathcal{W}^{10}_{2L\times L}\mathcal{W}^{10}_{L\times 2L},\end{aligned}$$

and the relation

$$\begin{aligned}&\mathcal{W}^{10}_{2L\times L}\left\{\mathcal{W}^{10}_{L\times 2L}\mathcal{P}_k(m+1)\mathcal{W}^{10}_{2L\times L}\right\}^{-1}\mathcal{W}^{10}_{L\times 2L} \\ &\quad = \mathcal{W}^{10}_{2L\times 2L}\mathcal{P}_k^{-1}(m+1)\end{aligned}$$

can be justified by post-multiplying both sides of the expression by $\mathcal{P}_k(m+1)\mathcal{W}^{10}_{2L\times L}$ and recognizing that $\mathcal{W}^{10}_{2L\times 2L}\mathcal{W}^{10}_{2L\times L} = \mathcal{W}^{10}_{2L\times L}$.

If the matrix $2\mathcal{W}^{10}_{2L\times 2L}$ is approximated by the identity matrix similar to (17), we finally deduce the *unconstrained* NMCFLMS algorithm:

$$\begin{aligned}&\underline{\boldsymbol{h}}_k^{10}(m+1) = \underline{\boldsymbol{h}}_k^{10}(m) - \\ &\quad \mu_f\mathcal{P}_k^{-1}(m+1)\sum_{i=1}^{M}\mathcal{D}^*_{x_i}(m+1)\underline{\boldsymbol{e}}^{01}_{ik}(m+1), \quad (20)\end{aligned}$$

where the normalization matrix $\mathcal{P}_k(m+1)$ is diagonal and it is straightforward to find its inverse. Again, the unit-norm constraint will be enforced on the model filter coefficients after every step of updation.

In the MCFLMS algorithm, the correction applied to the model filter in each update is approximately proportional to the power spectrum $\mathcal{P}_k(m+1)$; this can be seen by substituting (10) into (13) and using the approximation (17). When the channel outputs are large, gradient noise amplification may be experienced. With the normalization of the MCFLMS correction by $\mathcal{P}_k(m+1)$ in the NMCFLMS algorithm, this noise amplification problem is diminished and the variability of the convergence rates due to the change of signal level is eliminated. In order to estimate a more stable power spectrum, a recursive scheme is employed in implementation:

$$\begin{aligned}&\mathcal{P}_k(m+1) = \lambda\mathcal{P}_k(m) + \\ &\quad (1-\lambda)\sum_{i=1,i\neq k}^{M}\mathcal{D}^*_{x_i}(m+1)\mathcal{D}_{x_i}(m+1), \quad (21) \\ &\qquad k=1,2,...,M,\end{aligned}$$

where λ is a forgetting factor that may appropriately be set as $\lambda = [1 - 1/(3L)]^L$ for the NMCFLMS algorithm. Although the NMCFLMS algorithm bypasses the problem of noise amplification, we face a similar problem that occurs when the channel outputs becomes too small. An alternative, therefore, is to insert a small positive number δ into the normalization which leads to the following modification to the unconstrained NMCFLMS algorithm:

$$\begin{aligned}&\underline{\boldsymbol{h}}_k^{10}(m+1) = \underline{\boldsymbol{h}}_k^{10}(m) - \mu_f\left[\mathcal{P}_k(m) + \delta\mathbf{I}_{2L\times 2L}\right]^{-1}\cdot \\ &\quad \sum_{i=1}^{M}\mathcal{D}^*_{x_i}(m+1)\underline{\boldsymbol{e}}^{01}_{ik}(m+1),\ \ k=1,2,...,M. \quad (22)\end{aligned}$$

5. SIMULATIONS

To evaluate the performance of the proposed algorithm, we carried out Monte Carlo simulations for blind identification of a random three-channel SIMO system of order $L = 16$. For comparison, the cross relation (CR) batch method [4] and the time-domain multichannel Newton (MCN) algorithm [8] are also studied.

The normalized root mean square projection misalignment (NRMSPM) in dB is used as a performance measure of estimation accuracy in this paper and is given by

$$\text{NRMSPM} \triangleq 20\log_{10}\left[\frac{1}{\|\mathbf{h}_t\|}\sqrt{\frac{1}{N}\sum_{i=1}^{N}\|\epsilon^{(i)}\|^2}\right], \quad (23)$$

where N is the number of Monte-Carlo runs, $(\cdot)^{(i)}$ denotes a value obtained for the i-th run, and $\boldsymbol{\epsilon} = \mathbf{h}_t - [(\mathbf{h}_t^T\mathbf{h})/(\mathbf{h}^T\mathbf{h})]\hat{\mathbf{h}}$ is a projection error vector. By projecting $\mathbf{h}_t$ onto $\mathbf{h}$ and defining a projection error, we take only the misalignment of the channel estimate into account [10].

For a common floating-point implementation of an algorithm, the floating-point operations (flops) dominate the calculation and the number of flops is a consistent measure of the algorithm's computational complexity, independent of what machine it runs on. The flops per set of multi-channel outputs are counted. The absolute number of flops for the studied adaptive algorithms are not particularly meaningful, but their relative values illustrate the great efficiency of the frequency-domain approaches.

In the simulations, the source signal is an uncorrelated binary phase-shift-keying (BPSK) sequence. The additive noise is i.i.d. zero-mean Gaussian and the specified SNR is defined as SNR $\triangleq 10\log_{10}[\sigma_s^2\|\mathbf{h}_t\|^2/(M\sigma_b^2)]$, where σ_s^2 and σ_b^2 are the signal and noise powers, respectively.

A simple initialization was employed for all conducted experiments. In the time domain, the channel impulse response of the k-th channel is set as $\mathbf{h}_k(0) = [1/\sqrt{M}\ 0\ \cdots\ 0]^T, k = 1, 2, ..., M$. Since the initial channel estimates are identical, a non-zero error signal can be guaranteed and hence the channel filter coefficients will be properly adapted.

For the CR method, 120 samples of observations from each channel were utilized. For the MCN and MCFLMS algorithms, the step size $\rho = 0.95$ and $\mu_f = 4 \times 10^{-4}$ were fixed, respectively. For the NMCFLMS algorithm, the step size $\mu_f = 0.8$ was used and the regularization factor δ was initially set as one fifth of the total power over all channels at the first block. For each specified SNR value, the NRMSPM was calculated by averaging the results, after convergence, of $N = 200$ Monte Carlo runs.

As seen in Fig. 1, the NRMSPMs of all studied algorithms decrease steadily as the SNR increases. Fig. 2 shows the learning curves of these adaptive algorithms, among which the MCN algorithm converges fastest but, on the other hand, the variance of its cost function is also the largest after convergence. Although both the MCFLMS and NMCFLMS algorithms converge steadily to the desired channel impulse responses, apparently the NMCFLMS algorithm performs better, achieving a good compromise between fast convergence speed and low estimate variance. Fig. 3 gives a comparison of computational complexity among the investigated algorithms. Clearly, the frequency-domain approaches are much more efficient.

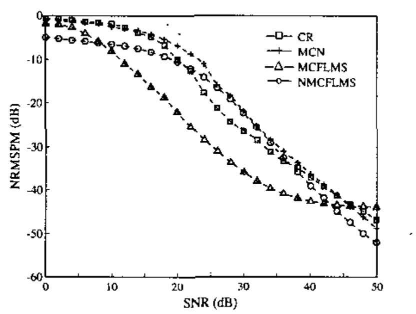

Figure 1: Comparison of converged NRMSPM vs. SNR among the CR, MCN, MCFLMS, and NMCFLMS algorithms for the random three-channel system excited by a random BPSK sequence.

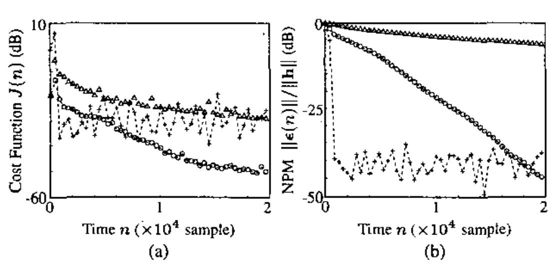

Figure 2: Comparison of convergence among the MCN ($- + -$), MCFLMS ($-\Delta-$), and NMCFLMS ($-\text{o}-$) algorithms for the random three-channel system, excited by a random BPSK sequence. Trajectories of (a) the cost function $J(n)$, and (b) the normalized projection misalignment (NPM) $\|\epsilon(n)\|/\|\mathbf{h}\|$ vs. time n are shown for one typical run of the three algorithms.

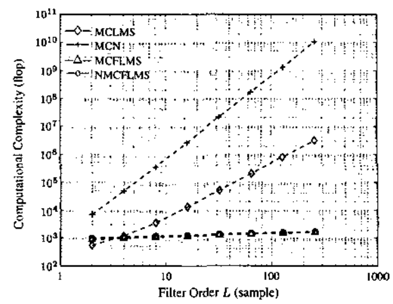

Figure 3: Comparison of computational complexity per set of multi-channel outputs among the MCLMS, MCN, MCFLMS, and NMCFLMS algorithms for the random three-channel system of different order (L), excited by a random BPSK sequence.

6. CONCLUSIONS

Blind identification of SIMO systems is examined and the issues of convergence, adaptivity, and efficiency of a satisfactory approach are addressed from a practical point of view. An adaptive algorithm using channel cross-relation is implemented in the frequency domain. As the experimental results supported, the proposed method achieves both fast convergence and great efficiency.

7. REFERENCES

[1] L. Tong, G. Xu, and T. Kailath, "A new approach to blind identification and equalization of multipath channels," in *Proc. 25th Asilomar Conf. on Signals, Systems, and Computers*, 1991, vol. 2, pp. 856–860.

[2] E. Moulines, P. Duhamel, J. F. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, no. 2, pp. 516–525, Feb. 1995.

[3] H. Liu, G. Xu, and L. Tong, "A deterministic approach to blind equalization," in *Proc. 27th Asilomar Conf. on Signals, Systems, and Computers*, 1993, vol. 1, pp. 751–755.

[4] G. Xu, H. Liu, L. Tong, and T. Kailath, "A least-squares approach to blind channel identification," *IEEE Trans. Signal Processing*, vol. 43, pp. 2982–2993, Dec. 1995.

[5] D. Slock, "Blind fractionally-spaced equalization, prefect reconstruction filerbanks, and multilinear prediction," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, 1994, vol. 4, pp. 585–588.

[6] G. Giannakis and J. Mendel, "Identification of nonminimum phase systems using higher order statistics," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. 37, pp. 7360-7377, Mar. 1989.

[7] L. Tong and S. Perreau, "Multichannel blind identification: from subspace to maximum likelihood methods," *Proc. IEEE*, vol. 86, no. 10, pp. 1951–1968, Oct. 1998.

[8] Y. Huang and J. Benesty, "Adaptive multi-channel least mean square and Newton algorithms for blind channel identification," *Signal Processing*, vol. 82/8, pp. 99–110, Aug. 2002.

[9] J. Benesty and D. Morgan, "Frequency-domain adaptive filtering revisited, generalization to the multi-channel case, and application to acoustic echo cancellation," in *Proc. IEEE Int. Conf. Acoust., Speech, Signal Processing*, 2000, vol. 2, pp. 789–792.

[10] D. R. Morgan, J. Benesty, and M. M. Sondhi, "On the evaluation of estimated impulse responses," *IEEE Signal Processing Lett.*, vol. 5, no. 7, pp. 174–176, Jul. 1998.