# SEPARATING ISI AND CCI IN A TWO-STEP FIR BEZOUT EQUALIZER FOR MIMO SYSTEMS OF FREQUENCY-SELECTIVE CHANNELS

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**ABSTRACT**

The use of multiple antennas at both the transmitter and the receiver in wireless communications implies a great channel capacity, but the signal detection is a crucial problem to achieve the channel capacity particularly in the general case of frequency selective channels, where both inter-symbol interference (ISI) and co-channel interference (CCI) are significant. In this paper, we show that ISI and CCI can be separated and then be cancelled in two different steps. We develop a two-step FIR Bezout equalizer and deduce the theoretically smallest length of its equalization filter.

## 1. INTRODUCTION

The capacity of wireless communication systems can be significantly enhanced by the use of multiple antennas at both transmission and reception, as shown by theoretical analyses [1], [2] and as evidenced by such an experimental multiple-input multiple-output (MIMO) system as the BLAST (Bell Laboratories Layered Space-Time) [3], [4]. The performance of signal detection algorithms developed for Rayleigh-faded channels is limited in many practical broadband wireless communication systems, where, in addition to co-channel interference (CCI), inter-symbol interference (ISI) can be severe. The need for efficient equalization techniques in frequency-selective MIMO systems is imperative and recently has gained considerable attention.

In the literature, MIMO equalizers were developed using either the MMSE (Minimum Mean Square Error) or the zero-forcing (also called Bezout) principles, and their architectures might be sequential with decision feedback or parallel otherwise. The sequential MMSE equalizer is optimum in the sense that it can achieve the full channel capacity with IIR filters [5]. A more practical FIR sequential MMSE equalizer was proposed in [6]. However, in order to achieve the capacity advantage of MMSE equalizers, an accurate SNR estimate is necessary and the filter length has to be decided empirically in *a priori*. Unfortunately these estimation and decision are not easy to make in practice. Therefore, a suboptimal while relatively simple Bezout equalizer turns out to be an appealing alternative. In this paper, we will demonstrate for the first time that the ISI and the CCI can be separated and then cancelled in two different steps. In the development of this novel two-step FIR Bezout equalizer, we will analytically illustrate when a MIMO system can be perfectly equalized or otherwise what would be the best achievable solution. In addition, we will derive the theoretically smallest length of equalization filters. Although a parallel structure is considered for ease of presentation, it should be straightforward to extend the idea to building a sequential equalizer.

## 2. SIGNAL MODEL

Consider an FIR $(M, N)$ MIMO system that consists of $M$ transmitting antennas and $N$ receiving antennas, with $M < N$. At the receiver $n$ and at the sample time $k$, we have:

$$x_n(k) = \sum_{m=1}^{M} \mathbf{h}_{nm}^T \mathbf{s}_m(k, L_h) + w_n(k), \quad k = 1, 2, \cdots, K, \;\; n = 1, 2, \cdots, N, \tag{1}$$

where $(\cdot)^T$ denotes the transpose of a matrix or a vector,

$$\mathbf{h}_{nm} = \begin{bmatrix} h_{nm,0} & h_{nm,1} & \cdots & h_{nm,L_h-1} \end{bmatrix}^T, \quad n = 1, 2, \cdots, N, \;\; m = 1, 2, \cdots, M,$$

is the complex impulse response (of length $L_h$, $\forall M, N$) between the transmitter $m$ and the receiver $n$, assumed to be constant for $K$ symbol periods,

$$\mathbf{s}_m(k, L_h) = [s_m(k) \;\; s_m(k-1) \;\; \cdots \;\; s_m(k - L_h + 1)]^T$$

is a vector containing the last $L_h$ samples of the source signal $s_m$, and $w_n(k)$ is a zero-mean complex additive white Gaussian noise (AWGN) with variance $\sigma_w^2$, $\forall n$.

Sometimes, it is more useful to use the $z$-transform of (1):

$$X_n(z) = \sum_{m=1}^{M} H_{nm}(z) S_m(z) + W_n(z), \tag{2}$$

where $H_{nm}(z) = \sum_{l=0}^{L_h-1} h_{nm,l} z^{-l}$.

In order to detect the transmitted symbols at the receivers, the complex channels $\mathbf{h}_{nm}$ need to be known. In practice, $\mathbf{h}_{nm}$ are identified by sending a training sequence at the beginning of each burst. In this paper, we assume that the system has been trained and we will make no distinction between $\mathbf{h}_{nm}$ and their estimates.

## 3. CONVERSION OF AN $(M, N)$ MIMO SYSTEMS TO $M$ SIMO SYSTEMS

In this section, we will explain how to separate the ISI and the CCI by converting an $(M, N)$ MIMO system into $M$ CCI-free SIMO systems. The development begins with an example of the simplest $(2, 3)$ MIMO system and then extends to a general $(M, N)$ case.

### 3.1. Example: From a $(2,3)$ MIMO System to Two SIMO Systems

For a $(2,3)$ MIMO system, the CCI can be cancelled by using two output signals at a time. For instance, we can remove the CCI in $X_1(z)$ and $X_2(z)$ caused by $S_2(z)$ as follows:

$$\begin{aligned} X_1(z)H_{22}(z) - X_2(z)H_{12}(z) = \\ [H_{11}(z)H_{22}(z) - H_{21}(z)H_{12}(z)]\, S_1(z) + \\ [H_{22}(z)W_1(z) - H_{12}(z)W_2(z)]\,. \end{aligned} \tag{3}$$

Similarly, the CCI caused by $S_1(z)$ from those two outputs can be cancelled. Therefore, by selecting different pairs from the three outputs, we could obtain six CCI-free signals and then could construct two separate single-input three-output systems with respect to two distinct inputs, respectively. This procedure is visualized in Fig. 1 and will be described in a more systematic way in the following.

Let us consider the following equation:

$$Y_{s_1,p}(z) = \sum_{q=1}^{3} H_{s_1,pq}(z)X_q(z), \;\; p = 1,2,3, \tag{4}$$

where $H_{s_1,pp}(z) = 0,\ \forall p$. This means that (4) considers only two receiving antenna signals for each $p$. The objective is to find the polynomials $H_{s_1,pq}(z)$, $p, q = 1, 2, 3$, $p \neq q$, in such a way that:

$$Y_{s_1,p}(z) = F_{s_1,p}(z)S_1(z) + W_{s_1,p}(z), \;\; p = 1,2,3, \tag{5}$$

which represents a SIMO system where $s_1$ is the transmitted signal, $y_{s_1,p}, p = 1,2,3$, are the received signals, $f_{s_1,p}$ are the corresponding complex paths, and $w_{s_1,p}$ is the noise at output $p$. As shown in Fig. 1, one possibility is to choose:

$$\begin{aligned} H_{s_1,12}(z) &= H_{32}(z), & H_{s_1,13}(z) &= -H_{22}(z), \\ H_{s_1,21}(z) &= H_{32}(z), & H_{s_1,23}(z) &= -H_{12}(z), \\ H_{s_1,31}(z) &= H_{22}(z), & H_{s_1,32}(z) &= -H_{12}(z). \end{aligned} \tag{6}$$

In this case, we find that:

$$\begin{aligned} F_{s_1,1}(z) &= H_{32}(z)H_{21}(z) - H_{22}(z)H_{31}(z), \\ F_{s_1,2}(z) &= H_{32}(z)H_{11}(z) - H_{12}(z)H_{31}(z), \\ F_{s_1,3}(z) &= H_{22}(z)H_{11}(z) - H_{12}(z)H_{21}(z). \end{aligned} \tag{7}$$

Since $\deg[H_{nm}(z)] = L_h - 1$, where $\deg[\cdot]$ is the degree of a polynomial, therefore $\deg[F_{s_1,p}(z)] \leq 2L_h - 2$. We can see from (7) that polynomials $F_{s_1,1}(z)$, $F_{s_1,2}(z)$, and $F_{s_1,3}(z)$ share common zeros if $H_{12}(z)$, $H_{22}(z)$, and $H_{32}(z)$ [or if $H_{11}(z)$, $H_{21}(z)$, and $H_{31}(z)$] share common zeros.

Now suppose that $C_2(z) = \gcd[H_{12}(z), H_{22}(z), H_{32}(z)]$, where $\gcd[\cdot]$ denotes the greatest common divisor of the polynomials involved. We have:

$$H_{n2}(z) = C_2(z)H'_{n2}(z), \;\; n = 1,2,3. \tag{8}$$

It is clear that the signal $s_2$ in (4) can be canceled by using the polynomials $H'_{n2}(z)$ [instead of $H_{n2}(z)$ as given in (6)], so that the SIMO system represented by (5) will change to:

$$Y'_{s_1,p}(z) = F'_{s_1,p}(z)S_1(z) + W'_{s_1,p}(z), \;\; p = 1,2,3, \tag{9}$$

where

$$F'_{s_1,p}(z)C_2(z) = F_{s_1,p}(z) \;\text{ and }\; W'_{s_1,p}(z)C_2(z) = W_{s_1,p}(z).$$

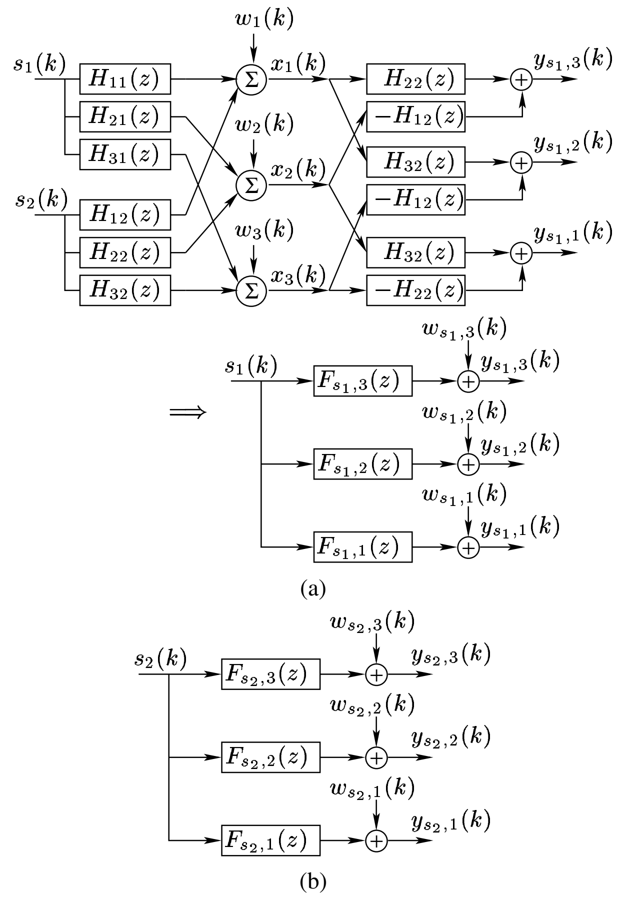


Figure 1: Illustration of conversion from a $(2,3)$ MIMO system to two CCI-free SIMO systems with respect to (a) $s_1(n)$ and (b) $s_2(n)$.

It is worth noticing that $\deg\left[F'_{s_1,p}(z)\right] \leq \deg[F_{s_1,p}(z)]$ and that polynomials $F'_{s_1,1}(z)$, $F'_{s_1,2}(z)$, and $F'_{s_1,3}(z)$ share common zeros if and only if $H_{11}(z)$, $H_{21}(z)$, and $H_{31}(z)$ share common zeros.

The second SIMO system corresponding to the source $s_2$ can be derived in a similar way and it can be shown that the two SIMO system (for $s_1$ and $s_2$) have identical channels but the noise at the receivers is different.

### 3.2. Generalization

The approach explained in the previous subsection on a simple example will be generalized here to an $(M, N)$ MIMO system ($M < N$). We begin with writing (2) into a vector form

$$\vec{\mathbf{X}}(z) = \mathbf{H}(z)\vec{\mathbf{S}}(z) + \vec{\mathbf{W}}(z), \tag{10}$$

where

$$\begin{aligned} \vec{\mathbf{X}}(z) &= [X_1(z)\;\; X_2(z)\;\; \cdots\;\; X_N(z)]^T\,, \\ \mathbf{H}(z) &= \begin{bmatrix} H_{11}(z) & H_{12}(z) & \cdots & H_{1M}(z) \\ H_{21}(z) & H_{22}(z) & \cdots & H_{2M}(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{N1}(z) & H_{N2}(z) & \cdots & H_{NM}(z) \end{bmatrix}, \end{aligned}$$

$$\vec{\mathbf{S}}(z) = [S_1(z)\ S_2(z)\ \cdots\ S_M(z)]^T,$$
$$\vec{\mathbf{W}}(z) = [W_1(z)\ W_2(z)\ \cdots\ W_N(z)]^T.$$

If $C_m(z) = \gcd[H_{1m}(z), H_{2m}(z), \cdots, H_{Nm}(z)]$ ($m$ = 1, 2, $\cdots$, $M$), then $H_{nm}(z) = C_m(z)H'_{nm}(z)$ and the channel matrix $\mathbf{H}(z)$ can be rewritten as

$$\mathbf{H}(z) = \mathbf{H}'(z)\mathbf{C}(z), \tag{11}$$

where $\mathbf{H}'(z)$ is an $N \times M$ matrix containing the elements $H'_{nm}(z)$ and $\mathbf{C}(z)$ is an $M \times M$ diagonal matrix with $C_m(z)$ as its nonzero components.

Let us choose $M$ from $N$ system outputs and we have $P = C_N^M$ different ways of doing this. For the $p$-th ($p = 1, 2, \cdots, P$) combination, we denote the index of the $M$ selected outputs as $p_m$, $m = 1, 2, \cdots, M$, and get an $(M, M)$ MIMO sub-system. Consider the following equations:

$$\vec{\mathbf{Y}}_p(z) = \mathbf{H}_{s,p}(z)\vec{\mathbf{X}}_p, \tag{12}$$

where

$$\vec{\mathbf{Y}}_p(z) = [Y_{s_1,p}(z)\ Y_{s_2,p}(z)\ \cdots\ Y_{s_M,p}(z)]^T,$$
$$\mathbf{H}_{s,p}(z) = \begin{bmatrix} H_{s_1,p1}(z) & H_{s_1,p2}(z) & \cdots & H_{s_1,pM}(z) \\ H_{s_2,p1}(z) & H_{s_2,p2}(z) & \cdots & H_{s_2,pM}(z) \\ \vdots & \vdots & \vdots & \vdots \\ H_{s_M,p1}(z) & H_{s_M,p2}(z) & \cdots & H_{s_M,pM}(z) \end{bmatrix},$$
$$\vec{\mathbf{X}}_p = \left[X_{s_{p_1}}(z)\ X_{s_{p_2}}(z)\ \cdots\ X_{s_{p_M}}(z)\right]^T.$$

Let $\mathbf{H}_p(z)$ be the $M \times M$ matrix obtained from $\mathbf{H}(z)$ by keeping its rows corresponding to the $M$ selected outputs. Substituting (10) into (12) yields

$$\vec{\mathbf{Y}}_p(z) = \mathbf{H}_{s,p}(z)\mathbf{H}_p(z)\vec{\mathbf{S}}(z) + \mathbf{H}_{s,p}(z)\vec{\mathbf{W}}_p(z). \tag{13}$$

In order to remove the CCI, the objective here is to find the matrix $\mathbf{H}_{s,p}(z)$ whose components are linear combinations of $H_{nm}(z)$ such that $\mathbf{H}_{s,p}(z)\mathbf{H}_p(z)$ would be a diagonal matrix. Consequently, we have

$$Y_{s_m,p}(z) = F_{s_m,p}(z)S_m(z) + W_{s_m,p}(z). \tag{14}$$

If $\mathbf{C}_p(z)$ [obtained from $\mathbf{C}(z)$ in a similar way as $\mathbf{H}_p(z)$ is constructed] is not equal to the identity matrix, then the CCI-free signals are determined as

$$\vec{\mathbf{Y}}'_p(z) = \mathbf{H}'_{s,p}(z)\mathbf{H}'_p(z)\mathbf{C}(z)\vec{\mathbf{S}}(z) + \mathbf{H}'_{s,p}(z)\vec{\mathbf{W}}_p(z), \tag{15}$$

and

$$Y'_{s_m,p}(z) = F'_{s_m,p}(z)S_m(z) + W'_{s_m,p}(z). \tag{16}$$

In rich scattering environments $\mathbf{H}'_p(z)$ has full column normal rank as assumed in this paper.

Obviously a good choice for $\mathbf{H}'_{s,p}(z)$ is the adjoint of matrix $\mathbf{H}'_p(z)$, i.e., the $(i,j)$-th element of $\mathbf{H}'_{s,p}(z)$ is the $(j,i)$-th cofactor of $\mathbf{H}'_p(z)$.

Since $F'_{s_m,p}(z) = \sum_{q=1}^{M} H'_{s_m,pq}(z)H_{p_q m}(z)$ and $H'_{s_m,pq}(z)$ ($q = 1, 2, \cdots, M$) are co-prime, the polynomials $F'_{s_m,p}(z)$ ($p = 1, 2, \cdots, P$) share common zeros if and only if the polynomials $H_{nm}(z)$ ($n = 1, 2, \cdots, N$) share common zeros. Therefore, if channels with respect to no more than one input share common zeros for an $(M, N)$ MIMO system, we can convert it into $M$ CCI-free SIMO systems whose $C_N^M$ channels are co-prime.

Also, it can easily be checked that $\deg\left[F'_{s_m,p}(z)\right] \le M(L_h - 1)$. As a result, the length of the FIR filter $f'_{s_m,p}$ would be

$$L_f \le M(L_h - 1) + 1. \tag{17}$$

## 4. EQUALIZATION

In the above, we showed that from an $(M, N)$ MIMO system we can derive $M$ CCI-free SIMO systems which are much easier to equalize than a MIMO system. The $M$ SIMO systems will be equalized in parallel.

For the SIMO system with respect to source $s_m$ ($m$ = 1, 2, $\cdots$, $M$), we consider the polynomials $G_{s_m,p}(z)$ ($p = 1, 2, \cdots, P$) and the equation:

$$\begin{aligned}\widehat{S}_m(z) &= \sum_{p=1}^{P} G_{s_m,p}(z)Y'_{s_m,p}(z) \\ &= \left[\sum_{p=1}^{P} F'_{s_m,p}(z)G_{s_m,p}(z)\right] S_m(z) + \\ &\quad \sum_{p=1}^{P} G_{s_m,p}(z)W'_{s_m,p}(z).\end{aligned} \tag{18}$$

The polynomials $G_{s_m,p}(z)$ are found in such a way that $\widehat{S}_m(z) = S_m(z)$ in the absence of noise by using the Bezout theorem which is:

$$\begin{aligned}&\gcd\left[F'_{s_m,1}(z), F'_{s_m,2}(z), \cdots, F'_{s_m,P}(z)\right] = 1 \\ &\Leftrightarrow\ \exists\, G_{s_m,1}(z), G_{s_m,2}(z), \cdots, G_{s_m,P}(z), \\ &\sum_{p=1}^{P} F'_{s_m,p}(z)G_{s_m,p}(z) = 1.\end{aligned} \tag{19}$$

The idea of using the Bezout theorem for equalization was first proposed in [7] in the context of room acoustics.

If the channels of the SIMO system share common zeros, i.e.,

$$C'_{s_m}(z) = \gcd\left[F'_{s_m,1}(z), F'_{s_m,2}(z), \cdots, F'_{s_m,P}(z)\right] \ne 1, \tag{20}$$

then we have

$$F'_{s_m,p}(z) = C'_{s_m}(z)F''_{s_m,p}(z),\ \ p = 1, 2, \cdots, P, \tag{21}$$

and the polynomials $G_{s_m,p}(z)$ can be found such that

$$\sum_{p=1}^{P} F''_{s_m,p}(z)G_{s_m,p}(z) = 1.$$

In this case the $m$-th SIMO system can be equalized up to the polynomial $C'_{s_m}(z)$. For complete equalization, we have to add another stage to the process by looking at $C'_{s_m}(z)$. If $C'_{s_m}(z)$ is minimum phase, a complete equalization still can be attained. Otherwise, a least squares solution is derived to at best minimize the effect of $C'_{s_m}(z)$.

To determine the equalizer, we write the Bezout equation in the time domain as:

$$\mathbf{F}'^{c}_{s_m}\mathbf{g}_{s_m} = \sum_{p=1}^{P} \mathbf{F}'^{c}_{s_m,p}\mathbf{g}_{s_m,p} = \mathbf{e}_1, \tag{22}$$

where

$$\mathbf{F}'^{c}_{s_m} = \left[\ \mathbf{F}'^{c}_{s_m,1}\ \ \mathbf{F}'^{c}_{s_m,2}\ \ \cdots\ \ \mathbf{F}'^{c}_{s_m,P}\ \right],$$
$$\mathbf{g}_{s_m} = \left[\ \mathbf{g}^T_{s_m,1}\ \ \mathbf{g}^T_{s_m,2}\ \ \cdots\ \ \mathbf{g}^T_{s_m,P}\ \right]^T,$$
$$\mathbf{g}_{s_m,p} = \left[\ g_{s_m,p,0}\ \ g_{s_m,p,1}\ \ \cdots\ \ g_{s_m,p,L_g-1}\ \right]^T,$$
$$m = 1, 2, \cdots, M,\ \ p = 1, 2, \cdots, P,$$

$L_g$ is the length of the FIR filter $g_{s_m,p}$,

$$\mathbf{F}'^{\,c}_{s_m,p} = \begin{bmatrix} f'_{s_m,p,0} & 0 & \cdots & 0 \\ f'_{s_m,p,1} & f'_{s_m,p,0} & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ f'_{s_m,p,L_f-1} & \cdots & \cdots & \vdots \\ 0 & f'_{s_m,p,L_f-1} & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & 0 & f'_{s_m,p,L_f-1} \end{bmatrix}$$

is an $(L_f + L_g - 1) \times L_g$ matrix, and $\mathbf{e}_1 = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$ is an $(L_f + L_g - 1) \times 1$ vector. In order to have a unique solution for (22), $L_g$ must be chosen in such a way that $\mathbf{F}'^{\,c}_{s_m}$ is a square matrix. In this case, we have

$$L_g = \frac{L_f - 1}{P - 1} \leq \frac{M(L_h - 1)}{P - 1}. \quad (23)$$

Although finding the shortest equalization filters involves the lowest computational complexity and leads to the most cost effective implementation, the performance may not be the best due to noise in practice. Moreover, the smallest $L_g$ may not be even realistic since (23) does not guarantee an integer solution. Therefore we choose a larger $L_g$ than necessary in our implementation and solve (22) for $\mathbf{g}_{s_m}$ in the least squares sense. Furthermore we allow a decision delay $d$ and replace $\mathbf{e}_1$ with $\mathbf{e}_d$ with all elements zero except 1 at position $d$.

## 5. SIMULATIONS

To evaluate the performance of the proposed two-step spatio-temporal equalization (TSSTE) algorithm, we present here a numerical study on an $(M = 2, N)$ MIMO system. For comparison, the parallel MMSE equalizer is also implemented.

In our experiment, we considered non-coded system performance and measured the bit error rate (BER) as a function of the SNR at each receiving antenna. The measurement was made over 2000 bursts. At the transmitters, each substream utilizes the 16-point rectangular QAM and every symbol takes four bits. In each substream of every burst, the first $4ML_h$ symbols are used for training and the following 100 symbols contain the effective information for transmission.

We have assumed that the additive noise at the receiver is i.i.d. complex white Gaussian with zero mean. The channel impulse responses $h_{nm}$ are of length $L_h = 4$. The coefficients are i.i.d. complex white Gaussian with zero mean and their variances, i.e., average energies, decay exponentially with tap index:

$$\sigma^2_{h_{nm,l}} \triangleq E\left\{\|h_{nm,l}\|^2\right\} = \exp(-\lambda l)/\Sigma_{nm}, \quad (24)$$
$$l = 0, 1, \cdots, L_h - 1, \ \forall m, n,$$

where $\lambda = 2$ is a decay constant and we use

$$\Sigma_{nm} = \sum_{l=0}^{L_h-1} \sigma^2_{h_{nm,l}}$$

to normalize the channel impulse responses making each channel have a unit total energy. In the experiment, the lengths of both the MMSE and the Bezout filters are set equal to $L_h$.

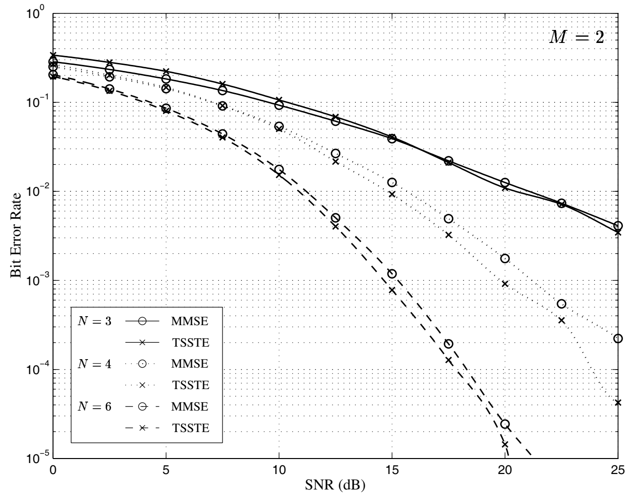


Figure 2: A performance comparison of BER vs. SNR between the MMSE and TSSTE algorithms for an $(2, N)$ MIMO system.

The experimental results are plotted in Fig. 2. Clearly the TSSTE achieves comparable performance as the MMSE and both produce less errors as more receiving antennas are used.

## 6. CONCLUSION

The inter-symbol and co-channel interference found in a MIMO system of frequency-selective channels can be separated and then be cancelled one after the other. This paper proposed a two-step procedure for spatio-temporal Bezout equalization and analyzed that under what conditions a perfect equalization could be realized or what would be the best achievable solution if these conditions cannot be met. In addition, a way to determine a proper length of the equalization filter was presented and what is the smallest length was discussed.

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