SUBSPACE SUPERDIRECTIVE BEAMFORMING WITH UNIFORM CIRCULAR MICROPHONE ARRAYS

Gongping Huang[1], *Jacob Benesty*[2], *and Jingdong Chen*[1]

[1]Center of Immersive and Intelligent Acoustics
Northwestern Polytechnical University
127 Youyi West Rd., Xi'an, Shaanxi 710072, China

[2]INRS-EMT, University of Quebec
800 de la Gauchetiere Ouest, Suite 6900
Montreal, QC H5A 1K6, Canada

ABSTRACT

Superdirective beamformers with uniform circular arrays (UCAs) are often used in communication applications for their steering flexibility and potential high directivity factors (DFs). This paper proposes two classes of subspace superdirective beamformers. The first class is based on the diagonalization of the noise pseudo-coherence matrix with the Fourier matrix while the second one is based on the joint diagonalization of the pseudo-coherence matrices of the desired and noise signals. By properly choosing the dimension of the subspace, the developed two classes of subspace superdirective beamformers can find good compromises between large DFs and reasonable values of white noise gains, which make them much more flexible than the conventional regularized superdirective beamformer.

Index Terms— Uniform circular arrays (UCAs), superdirective beamforming, robust beamforming, supergain, white noise gain, directivity factor, joint diagonalization, subspace.

1. INTRODUCTION

Microphone array systems have been widely used in speech and audio applications to extract signals of interest from noisy observations in room acoustic environments [1–11]. One of the core components in a microphone array system is the so-called beamforming, which aims at recovering a desired signal from noisy observations with a spatial filter [12–15]. Circular arrays have drawn a significant amount of attention since, thanks to their symmetries, they can have a similar performance in different steering directions [16–18]. Similarly to linear arrays, many beamforming algorithms have been developed, such as the delay-and-sum, filter-and-sum, and superdirective beamformers [1, 19–24].

In applications where small-size microphone arrays are required, superdirective beamforming, which achieves maximum gain in a diffuse noise field, is very attractive [1, 25–28]. However, superdirective beamformers may suffer from the problem of white noise amplification, making them sensitive to the array and sensors' imperfections. This is indeed the main problem that prevents superdirective beamformers from being widely used in practical systems [25, 29]. Therefore, how to achieve a relatively high directivity factor (DF) with a reasonable white noise gain (WNG) is becoming an important issue in the design of superdirective beamformers. Much effort has been devoted to solving this problem and the most common solution so far is the regularized superdirective beamformer [23, 25, 30], which introduces a WNG constraint to improve robustness. The performance of the regularized superdirective beamformer is controlled by a regularization parameter [13, 23, 25]. In practice, however, it is not always convenient to tune this parameter for a large DF and a reasonable value of WNG. So, other alternatives should be developed.

This paper presents two classes of subspace superdirective beamformers to deal with the white noise amplification problem. The first one is based on the decomposition the noise pseudo-coherence matrix, where the conventional superdirective beamformer can be written as a combination of a set of subfilters [16, 31, 32]. To improve robustness, we consider the beamformer which only uses the subfilters corresponding to the first few largest eigenvalues. The second class is based on the joint diagonalization of the pseudo-coherence matrices of the desired and noise signals [19]. With an M-sensor array, the joint diagonalization matrix constitutes an M dimensional space, where the first vector (with the maximum eigenvalue) corresponds to the signal subspace and the remaining $M-1$ vectors (with all the zero eigenvalues) correspond to the noise subspace. While the regular superdirective beamformer only uses the first vector corresponding to the largest eigenvalue, the remaining $M-1$ vectors can be used to improve the robustness of the superdirective beamformer.

2. SIGNAL MODEL AND PROBLEM FORMULATION

We consider a uniform circular array (UCA), of radius r, consisting of M omnidirectional microphones. We assume that a source signal impinges on the UCA from the farfield (plane wave) at the speed of sound, i.e., $c = 340$ m/s. We want to steer the main beam to the direction θ. In this scenario, the steering vector of length M is [6]

$$\mathbf{d}(\omega,\theta) = \Big[e^{\jmath\omega r c^{-1}\cos(\theta-\psi_1)} \quad e^{\jmath\omega r c^{-1}\cos(\theta-\psi_2)} \quad \cdots \quad e^{\jmath\omega r c^{-1}\cos(\theta-\psi_M)} \Big]^T, \quad (1)$$

where the superscript ^{T} is the transpose operator, $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\omega = 2\pi f$ is the angular frequency, $f > 0$ is the temporal frequency, and $\psi_m = [2\pi(m-1)]/M$ is the angular position of the mth $(m = 1, 2, \ldots, M)$ array element.

Assuming that the desired signal comes from the direction $\theta_{\rm s}$, the frequency-domain signal received by the mth $(m = 1, 2, \ldots, M)$ microphone is

$$Y_m(\omega) = e^{\jmath\omega r c^{-1}\cos(\theta_{\rm s}-\psi_m)} X(\omega) + V_m(\omega), \quad (2)$$

where $X(\omega)$ is the desired signal and $V_m(\omega)$ is the additive noise

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at the mth microphone. In a vector notation, (2) becomes

$$\begin{aligned}\mathbf{y}(\omega) &= \begin{bmatrix} Y_1(\omega) & Y_2(\omega) & \cdots & Y_M(\omega) \end{bmatrix}^T \\ &= \mathbf{x}(\omega) + \mathbf{v}(\omega) \\ &= \mathbf{d}(\omega, \theta_{\rm s}) X(\omega) + \mathbf{v}(\omega),\end{aligned} \tag{3}$$

where $\mathbf{d}(\omega, \theta_{\rm s})$ is the steering vector at $\theta = \theta_{\rm s}$, $\mathbf{x}(\omega) = \mathbf{d}(\omega, \theta_{\rm s}) X(\omega)$, and $\mathbf{v}(\omega)$ is defined similarly to $\mathbf{y}(\omega)$.

The objective of beamforming is to recover the desired signal, $X(\omega)$, from the noisy observation vector, $\mathbf{y}(\omega)$. To achieve that, a complex weight, $H_m^*(\omega)$, where the superscript * stands for complex conjugation, is applied at the output of each microphone. The weighted outputs are then summed together to get an estimate of the desired signal [4], i.e.,

$$Z(\omega) = \sum_{m=1}^{M} H_m^*(\omega) Y_m(\omega) = \mathbf{h}^H(\omega) \mathbf{y}(\omega), \tag{4}$$

where

$$\mathbf{h}(\omega) = \begin{bmatrix} H_1(\omega) & H_2(\omega) & \cdots & H_M(\omega) \end{bmatrix}^T \tag{5}$$

is a spatial filter of length M, and the superscript ^{H} is the transpose-conjugate operator.

3. PERFORMANCE MEASURES

The beampattern or directivity pattern describes the sensitivity of the beamformer to a plane wave (source signal) impinging on the UCA from the direction θ. Mathematically, it is defined as

$$\mathcal{B}[\mathbf{h}(\omega), \theta] = \mathbf{h}^H(\omega) \mathbf{d}(\omega, \theta). \tag{6}$$

Without loss of generality, we consider the first microphone as the reference. The input signal-to-noise ratio (SNR) is then defined as

$$\text{iSNR}(\omega) = \frac{\phi_X(\omega)}{\phi_{V_1}(\omega)}, \tag{7}$$

where $\phi_X(\omega) = E[|X(\omega)|^2]$ and $\phi_{V_1}(\omega) = E[|V_1(\omega)|^2]$ are the variances of $X(\omega)$ and $V_1(\omega)$, respectively. The output SNR is [17]

$$\text{oSNR}[\mathbf{h}(\omega)] = \frac{\phi_X(\omega)}{\phi_{V_1}(\omega)} \times \frac{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\mathbf{x}}(\omega) \mathbf{h}(\omega)}{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\mathbf{v}}(\omega) \mathbf{h}(\omega)}, \tag{8}$$

where $\mathbf{\Gamma}_{\mathbf{x}}(\omega) = \mathbf{d}(\omega, \theta_{\rm s}) \mathbf{d}^H(\omega, \theta_{\rm s})$ is the pseudo-coherence matrix of $\mathbf{x}(\omega)$, and $\mathbf{\Phi}_{\mathbf{v}}(\omega) = E\left[\mathbf{v}(\omega) \mathbf{v}^H(\omega)\right]$ and $\mathbf{\Gamma}_{\mathbf{v}}(\omega) = \frac{\mathbf{\Phi}_{\mathbf{v}}(\omega)}{\phi_{V_1}(\omega)}$ are the correlation and pseudo-coherence matrices of $\mathbf{v}(\omega)$, respectively.

The definition of the gain in SNR is easily derived from the previous definitions, i.e.,

$$\mathcal{G}[\mathbf{h}(\omega)] = \frac{\text{oSNR}[\mathbf{h}(\omega)]}{\text{iSNR}(\omega)} = \frac{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\mathbf{x}}(\omega) \mathbf{h}(\omega)}{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\mathbf{v}}(\omega) \mathbf{h}(\omega)}. \tag{9}$$

In our discussion, we are interested in two types of noise.

- The temporally and spatially white noise with the same variance at all microphones[1]. In this case, $\mathbf{\Gamma}_{\mathbf{v}}(\omega) = \mathbf{I}_M$, where $\mathbf{I}_M$ is the $M \times M$ identity matrix. Therefore, the gain in SNR is

$$\mathcal{W}[\mathbf{h}(\omega)] = \frac{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\mathbf{x}}(\omega) \mathbf{h}(\omega)}{\mathbf{h}^H(\omega) \mathbf{h}(\omega)},$$

  which is called the white noise gain (WNG).

- The diffuse noise[2]. In this case, the noise pseudo-coherence matrix is circulant (and symmetric) and can be written as

$$\begin{aligned}\mathbf{\Gamma}_{\mathbf{v}}(\omega) &= \mathbf{\Gamma}_{\rm d}(\omega) \\ &= \begin{bmatrix} \rho_1(\omega) & \rho_2(\omega) & \cdots & \rho_M(\omega) \\ \rho_M(\omega) & \rho_1(\omega) & \cdots & \rho_{M-1}(\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \rho_2(\omega) & \rho_3(\omega) & \cdots & \rho_1(\omega) \end{bmatrix},\end{aligned} \tag{10}$$

  where $\rho_m(\omega) = \text{sinc}(\omega \delta_m / c)$ with $\delta_m = 2r \sin[(m-1)\pi/M]$ being the distance between the reference and mth microphones. Now, the gain in SNR is

$$\mathcal{D}[\mathbf{h}(\omega)] = \frac{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\mathbf{x}}(\omega) \mathbf{h}(\omega)}{\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\rm d}(\omega) \mathbf{h}(\omega)}, \tag{11}$$

  which is called the directivity factor (DF) [3].

4. SUPERDIRECTIVE BEAMFORMING

In this section, we study superdirective and subspace superdirective beamformers with UCAs. Let us consider $\theta_{\rm s} = 0$ (the results can be easily generalized to other cases) and write $\mathbf{d}(\omega, 0) = \mathbf{d}(\omega)$ in the rest. In our context, the distortionless constraint at the desired direction $\theta_{\rm s} = 0$ is desired, i.e.,

$$\mathcal{B}[\mathbf{h}(\omega), 0] = \mathbf{h}^H(\omega) \mathbf{d}(\omega) = 1. \tag{12}$$

4.1. Conventional Superdirective Beamforming

By minimizing the residual noise, i.e., $\mathbf{h}^H(\omega) \mathbf{\Gamma}_{\rm d}(\omega) \mathbf{h}(\omega)$ subject to the distortionless constraint, we obtain the conventional superdirective beamformer [1]:

$$\mathbf{h}_{\rm S}(\omega) = \frac{\mathbf{\Gamma}_{\rm d}^{-1}(\omega) \mathbf{d}(\omega)}{\mathbf{d}^H(\omega) \mathbf{\Gamma}_{\rm d}^{-1}(\omega) \mathbf{d}(\omega)}. \tag{13}$$

In fact, it can be shown that (13) is the hypercardioid of order $\mathcal{M} - 1$ [17], where $\mathcal{M} = \lfloor \frac{M}{2} \rfloor + 1$ with $\lfloor x \rfloor$ being the integral part of x.

It is well known that the superdirective beamformer is very sensitive to small errors in the system. To deal with this problem, a robust version of the superdirective beamformer was derived in [13], which is

$$\mathbf{h}_{{\rm S},\epsilon}(\omega) = \frac{\left[\mathbf{\Gamma}_{\rm d}(\omega) + \epsilon \mathbf{I}_M\right]^{-1} \mathbf{d}(\omega)}{\mathbf{d}^H(\omega) \left[\mathbf{\Gamma}_{\rm d}(\omega) + \epsilon \mathbf{I}_M\right]^{-1} \mathbf{d}(\omega)}, \tag{14}$$

where $\epsilon \geq 0$ is a Lagrange multiplier that can be adjusted to satisfy the WNG constraint. This parameter ϵ tries to find a good compromise between a supergain and white noise amplification. It is clear that (14) is a regularized version of (13), where ϵ can be viewed as the regularization parameter. However, in practice, it is not very easy to determine the optimal value of this parameter.

4.2. Subspace Superdirective Beamforming based on the Noise Pseudo-Coherence Matrix Decomposition

With UCAs, the noise pseudo-coherence matrix is circulant and can be decomposed as [16, 32]

$$\mathbf{\Gamma}_{\rm d}(\omega) = \mathbf{F} \mathbf{\Lambda}(\omega) \mathbf{F}^H, \tag{15}$$

[1]This noise models well the sensor noise.

[2]This situation corresponds to the spherical isotropic noise field.

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{f}_1 & \mathbf{f}_2 & \cdots & \mathbf{f}_M \end{bmatrix} \quad (16)$$

is the $M \times M$ Fourier matrix,

$$\mathbf{f}_m = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & e^{\jmath(m-1)\beta} & \ldots & e^{\jmath(m-1)(M-1)\beta} \end{bmatrix}^T, \quad (17)$$

with $\beta = 2\pi/M$, and

$$\mathbf{\Lambda}(\omega) = \text{diag} \begin{bmatrix} \lambda_1(\omega), \lambda_2(\omega), \ldots, \lambda_M(\omega) \end{bmatrix} \quad (18)$$

is an $M \times M$ diagonal matrix, with

$$\lambda_m(\omega) = \sum_{i=1}^{M} \rho_m(\omega) e^{\jmath(i-1)(m-1)\beta}. \quad (19)$$

The eigenvectors and eigenvalues have the following symmetry [16]:

$$\begin{aligned} \mathbf{f}_{i+1} &= \mathbf{f}^*_{M-i+1}, \\ \lambda_{i+1}(\omega) &= \lambda_{M-i+1}(\omega), \end{aligned} \quad (20)$$

where $i = 0, 1, ..., \mathcal{M}-1$ ($\mathcal{M} = \lfloor \frac{M}{2} \rfloor + 1$) and $\lambda_1(\omega) \geq \lambda_2(\omega) \geq \cdots \geq \lambda_{\mathcal{M}}(\omega) > 0$.

Based on this decomposition, an analytical and closed-form expression of the superdirective beamformer was proposed in [16]. Following the same line of principles, we develop, in this part, a subspace superdirective beamformer. The conventional superdirective beamformer can be rewritten as

$$\begin{aligned} \mathbf{h}_{\text{S}}(\omega) &= \sum_{i=1}^{\mathcal{M}} \frac{\epsilon_i}{\lambda_i(\omega)} \left(\mathbf{f}_i \mathbf{f}_i^H + \mathbf{f}_i^* \mathbf{f}_i^T \right) \mathbf{d}(\omega) \\ &= \sum_{i=1}^{\mathcal{M}} \zeta_i(\omega) \mathbf{f}'_i(\omega), \end{aligned} \quad (21)$$

where $\epsilon_1 = 1$, $\epsilon_i = 2$ ($m = 2, \ldots, \mathcal{M}-1$), $\epsilon_{\mathcal{M}} = 1$ if $\mathcal{M} = M/2 + 1$, $\epsilon_{\mathcal{M}} = 2$ if $\mathcal{M} = (M+1)/2$, and

$$\zeta_i(\omega) = \frac{\epsilon_i}{\lambda_i(\omega)}, \quad (22)$$

$$\mathbf{f}_i{}'(\omega) = \Re\left(\mathbf{f}_i \mathbf{f}_i^H \right) \mathbf{d}(\omega), \quad (23)$$

with $\Re(\cdot)$ being the real part of a complex number, and we call $\mathbf{f}_i{}'(\omega)$ a subfilter. Clearly, the conventional superdirective beamformer consists of a combination of subfilters, where the eigenvalue $\lambda_i(\omega)$ in the denominator of (22) plays an important role in the performance of the beamformer. A smaller value of $\lambda_i(\omega)$ will have a more impact on the beamformer's robustness [16].

To improve WNG, we consider to use only N subfilters with $N \leqslant \mathcal{M}$ and construct a subspace superdirective beamformer as

$$\mathbf{h}_N(\omega) = \kappa(\omega) \sum_{i=1}^{N} \zeta_i(\omega) \mathbf{f}'_i(\omega) = \kappa(\omega) \mathbf{F}'_N(\omega) \boldsymbol{\zeta}_N(\omega), \quad (24)$$

where $\kappa(\omega)$ is a constant to meet the distortionless constraint,

$$\boldsymbol{\zeta}_N(\omega) = \begin{bmatrix} \zeta_1(\omega) & \zeta_2(\omega) & \cdots & \zeta_N(\omega) \end{bmatrix}^T \quad (25)$$

is a vector of length N, and

$$\mathbf{F}'_N(\omega) = \begin{bmatrix} \mathbf{f}'_1(\omega) & \mathbf{f}'_2(\omega) & \cdots & \mathbf{f}'_N(\omega) \end{bmatrix} \quad (26)$$

is a matrix of size $M \times N$.

Substituting (24) into the distortionless constraint, we get the first class of subspace superdirective beamformers:

$$\mathbf{h}_N(\omega) = \frac{\mathbf{F}'_N(\omega) \boldsymbol{\zeta}_N(\omega)}{\mathbf{d}^H(\omega) \mathbf{F}'_N(\omega) \boldsymbol{\zeta}_N(\omega)}. \quad (27)$$

Clearly, the performance of $\mathbf{h}_N(\omega)$ is strongly influenced by the dimension of the subspace. By playing with the value of N, we can obtain different beamformers.

- For $N = \mathcal{M}$, we get the conventional superdirective beamformer: $\mathbf{h}_{\mathcal{M}}(\omega) = \mathbf{h}_{\text{S}}(\omega)$.
- For $N = 1$, we obtain an averaging filter: $\mathbf{h}_1(\omega) \propto \begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$. Strictly speaking, this filter is not a beamformer with a UCA as it cannot produce a main beam. To produce a main beam along the source direction, we have to take $N \geq 2$.
- For $2 < N < \mathcal{M}$, we obtain beamformers who have higher WNG but lower DF than the conventional superdirective beamformer.

4.3. Subspace Superdirective Beamforming based on Joint Diagonalization

Alternately, we can develop a subspace superdirective beamformer based on the joint diagonalization of the desired and noise signals pseudo-coherence matrices [19]. The two Hermitian matrices $\mathbf{\Gamma}_{\mathbf{x}}(\omega)$ and $\mathbf{\Gamma}_{\text{d}}(\omega)$ can be jointly diagonalized as [19, 33]:

$$\mathbf{B}^H(\omega) \mathbf{\Gamma}_{\mathbf{x}}(\omega) \mathbf{B}(\omega) = \mathbf{\Lambda}'(\omega), \quad (28)$$

$$\mathbf{B}^H(\omega) \mathbf{\Gamma}_{\text{d}}(\omega) \mathbf{B}(\omega) = \mathbf{I}_M, \quad (29)$$

where

$$\mathbf{B}(\omega) = \begin{bmatrix} \mathbf{b}_1(\omega) & \mathbf{b}_2(\omega) & \cdots & \mathbf{b}_M(\omega) \end{bmatrix} \quad (30)$$

is an $M \times M$ full-rank square matrix and

$$\mathbf{\Lambda}'(\omega) = \text{diag} \begin{bmatrix} \lambda'_1(\omega), 0, \ldots, 0 \end{bmatrix} \quad (31)$$

is an $M \times M$ diagonal matrix.

It is easy to verify that the only nonnull eigenvalue is

$$\lambda'_1(\omega) = \mathbf{d}^H(\omega) \mathbf{\Gamma}_{\text{d}}^{-1}(\omega) \mathbf{d}(\omega), \quad (32)$$

whose corresponding eigenvector is

$$\mathbf{b}_1(\omega) = \frac{\mathbf{\Gamma}_{\text{d}}^{-1}(\omega) \mathbf{d}(\omega)}{\sqrt{\mathbf{d}^H(\omega) \mathbf{\Gamma}_{\text{d}}^{-1}(\omega) \mathbf{d}(\omega)}}. \quad (33)$$

It is clearly seen that the conventional superdirective beamformer is also

$$\mathbf{h}_{\text{S}}(\omega) = \frac{1}{\sqrt{\lambda'_1(\omega)}} \mathbf{b}_1(\omega), \quad (34)$$

which only uses the first eigenvector.

We observe that the joint diagonalizing matrix, $\mathbf{B}(\omega)$, constitutes an M dimensional space, where the first vector (with the maximum eigenvalue) corresponds to the signal subspace and the remaining $M-1$ vectors (with zero eigenvalues) correspond to the noise subspace. While the conventional superdirective beamformer only uses the first vector, the remaining $M-1$ vectors can be used to improve the robustness of the beamformer. We consider subspace superdirective beamformers with the following form:

$$\mathbf{h}_N(\omega) = \sum_{n=1}^{N} A_n \mathbf{b}_n(\omega) = \mathbf{B}_N(\omega)\,\mathbf{a}_N(\omega), \tag{35}$$

where A_n, $n = 1, 2, \ldots, N$ $(1 \le N \le M)$, are arbitrary numbers with at least one of them different from 0,

$$\mathbf{a}_N(\omega) = \begin{bmatrix} A_1(\omega) & A_2(\omega) & \cdots & A_N(\omega) \end{bmatrix}^T \tag{36}$$

is a vector of length N, and

$$\mathbf{B}_N(\omega) = \begin{bmatrix} \mathbf{b}_1(\omega) & \mathbf{b}_2(\omega) & \cdots & \mathbf{b}_N(\omega) \end{bmatrix} \tag{37}$$

is a matrix of size $M \times N$, which contains the first N columns of $\mathbf{B}(\omega)$.

With the beamformer given in (35), the WNG can be expressed as

$$\mathcal{W}[\mathbf{h}_N(\omega)] = \frac{\left|\mathbf{a}_N^H(\omega)\,\mathbf{B}_N^H(\omega)\,\mathbf{d}(\omega)\right|^2}{\mathbf{a}_N^H(\omega)\,\mathbf{B}_N^H(\omega)\,\mathbf{B}_N(\omega)\,\mathbf{a}_N(\omega)}. \tag{38}$$

Maximizing the WNG subject to the distortionless constraint and substituting the result into (35), we get the second class of subspace superdirective beamformers:

$$\mathbf{h}_N(\omega) = \frac{\mathbf{P}_{\mathbf{B}_N}(\omega)\,\mathbf{d}(\omega)}{\mathbf{d}^H(\omega)\,\mathbf{P}_{\mathbf{B}_N}(\omega)\,\mathbf{d}(\omega)}, \tag{39}$$

where

$$\mathbf{P}_{\mathbf{B}_N}(\omega) = \mathbf{B}_N(\omega)\left[\mathbf{B}_N^H(\omega)\,\mathbf{B}_N(\omega)\right]^{-1}\mathbf{B}_N^H(\omega). \tag{40}$$

The performance of $\mathbf{h}_N(\omega)$ is also strongly influenced by the dimension of the subspace. By playing with the value of N, we obtain different beamformers.

- For $N = 1$, we obtain the conventional superdirective beamformer: $\mathbf{h}_1(\omega) = \mathbf{h}_\mathrm{S}(\omega)$.
- For $N = M$, we obtain the delay-and-sum (DS) beamformer: $\mathbf{h}_M(\omega) = \mathbf{h}_\mathrm{DS}(\omega) = \frac{1}{M}\mathbf{d}(\omega)$.
- For $1 < N < M$, we obtain beamformers whose DFs and WNGs are in between those of the conventional superdirective and DS beamformers.

5. SIMULATIONS

In simulations, we consider a UCA consisting of eight closely spaced microphones, with r = 3 cm. The desired signal impinges on the array from the incidence angle $\theta_\mathrm{s} = 0$.

The WNG and the DF of the first class of subspace superdirective beamformers (with $N = 2, 3, 4, 5$) are plotted in Fig. 1 as a function of frequency. It is seen that, as the value of N increases from 2 to 5, the DF increases while the value of the WNG decreases. For $N = 5$, we get the conventional superdirective beamformer, which has a high DF but suffers from significant white noise amplification, particularly at low frequencies. So, the compromise between a good DF and a reasonable value of the WNG can be achieved by choosing a proper value of N.

The WNG and the DF of the second class of subspace superdirective beamformers (with $N = 1, 3, 5, 8$) are plotted in Fig. 2. For $N = 1$, this subspace superdirective beamformer degenerates to the conventional superdirective beamformer. For $N = 8$, it becomes the DS beamformer, which has the maximum WNG but its DF is small. Similar to the first class of subspace superdirective beamformers, this second class can also achieve a relatively high DF with a reasonable value of the WNG by choosing a proper value of N.

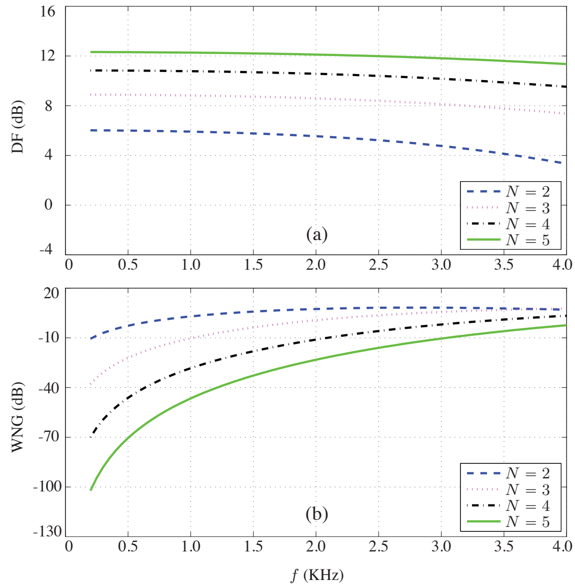

Fig. 1. Performance of the first class of subspace superdirective beamformers: (a) DF and (b) WNG. $r = 3$ cm, $M = 8$, and $\theta_\mathrm{s} = 0$.

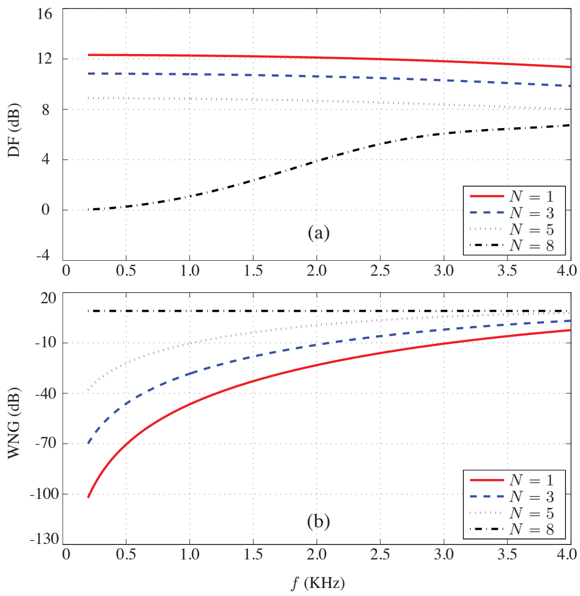

Fig. 2. Performance of the second class of subspace superdirective beamformers: (a) DF and (b) WNG. $r = 3$ cm, $M = 8$, and $\theta_\mathrm{s} = 0$.

6. CONCLUSIONS

Superdirective beamforming has been intensively studied for its ability to achieve high directivity factors (DFs). However, superdirective beamformers are found to be sensitive to sensors' noise and mismatch between sensors, particularly at low frequencies. This paper studied the problem of robust superdirective beamforming with uniform circular arrays (UCAs). We developed two classes of subspace superdirective beamformers. By choosing properly the dimension of the subspace, the developed beamformers can achieve a good compromise between a large DF and a reasonable value of the white noise gain (WNG). Simulation results demonstrated the good properties of the proposed subspace superdirective beamformers.

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