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***Abstract*—In this paper, we address the problem of designing broadband superdirective beamformers under some uncertainties in the direction of arrival (DoA) of the desired source. We formulate the signal model with a random steering vector, and propose a set of maximum signal-to-noise ratio gain (MSNRG) beamformers from the perspective of maximizing the signal-to-noise ratio (SNR) gain. The MSNRG beamformers can achieve maximum SNR gain and almost frequency-invariant beampatterns. Preliminary simulation results illustrate the properties and advantages of the proposed beamformers.**

***Index Terms*—Microphone arrays, fixed beamforming, superdirective beamforming, random steering vector, signal-to-noise ratio (SNR) gain.**

I. Introduction

Beamforming is one of the the most popular methods to deal with ubiquitous background noise, reverberation, competing sources, and interferences [1]–[3]. Beamforming algorithms can be broadly divided into two categories [2], [4], i.e., fixed and adaptive ones. In this paper, we focus on the design of fixed beamformers.

The delay-and-sum (DS) beamforming structure, which has been widely used in narrowband applications [5], generally has frequency-dependent directivity patterns [6]. The beamwidth of the main lobe of the DS beamformer decreases with the increase of frequency, which makes it unsuitable for processing broadband desired signals, such as speech, and suppressing noise and interferences at low frequency bands. Another drawback of this beamformer is its low directivity factor (DF), which limits its performance in dealing with reverberation. Recently, superdirective (SD) [7]–[10] and differential [11]–[16] beamformers have attracted much attention due to their frequency-invariant beampatterns, high DFs, and compact structures. Traditionally, most of the SD and differential beamformers are designed for point sources with fixed directions. In practice, however, there are some uncertainties in the DoA of the source of interest due to multiple reasons such as movement of the source, inaccurate DoA estimation, etc [17]–[20]. To take into account DoA uncertainties, we reformulate the signal model in this paper and develop a set of MSNRG beamformers that yield the maximum SNR gain. The beampatterns obtained with the MSNRG beamformers are almost frequency invariant when they are applied to microphone arrays with small interelement spacing. The MSNRG beamformers can control the beamwidth of the main lobe indirectly. Preliminary simulation results illustrate the properties and advantages of the proposed MSNRG beamformers.

II. Signal Model and Problem Formulation

We consider a plane wave, in the farfield, that propagates in an anechoic acoustic environment at the speed of sound, i.e., $c = 340$ m/s, and impinges on a uniform linear sensor array consisting of M omnidirectional microphones, where the distance between two successive sensors is equal to δ. The direction of the source signal to the array is parameterized by the azimuth angle θ. In this context, the steering vector (of length M) is given by

$$\mathbf{d}(\omega, \theta) = \begin{bmatrix} 1 & e^{-\jmath\omega\tau_0\cos\theta} & \cdots & e^{-\jmath(M-1)\omega\tau_0\cos\theta} \end{bmatrix}^T, \quad (1)$$

where the superscript ^{T} is the transpose operator, $\jmath = \sqrt{-1}$ is the imaginary unit, $\omega = 2\pi f$ is the angular frequency, $f > 0$ is the temporal frequency, and $\tau_0 = \delta/c$ is the time delay between two successive sensors at the angle $\theta = 0$.

We are interested in fixed beamformers with small values of δ, which is much smaller than $\lambda/2$, where λ is the signal wavelength, like in superdirective [7], [21] or differential [22], [23] beamforming, in order to eliminate spatial aliasing and achieve high DFs and frequency-independent beampatterns. But, contrary to these beamformers, the source of interest (desired source) may not be exactly at the angle $\theta_{\mathrm{s}} = 0$ (endfire direction) in our context. Here, we assume that there are some uncertainties in the desired source direction θ_{s}, so that its corresponding steering vector, $\mathbf{d}(\omega, \theta_{\mathrm{s}})$, is random. To simplify the derivation of the beamformers, it will be assumed in the rest of this paper that θ_{s} is uniformly distributed on the interval $[0, \psi]$, where ψ $(0 < \psi < \pi)$ is a design parameter. In practice, ψ can be estimated by the algorithms presented in [24]–[26]. For conciseness, we assume that the microphone array and desired source are in the same plane and the elevation angle is 0. In this case, the mean and the

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covariance matrix of $\mathbf{d}(\omega, \theta_\mathrm{s})$ are, respectively,

$$\begin{aligned}\overline{\mathbf{d}}_\psi(\omega) &= E\left[\mathbf{d}(\omega, \theta_\mathrm{s})\right] \\ &= \frac{1}{\psi}\int_0^\psi \mathbf{d}(\omega, \theta_\mathrm{s})\, d\theta_\mathrm{s} \end{aligned} \tag{2}$$

and

$$\begin{aligned}\mathbf{\Phi}_{\mathbf{d},\psi}(\omega) &= E\left\{\left[\mathbf{d}(\omega, \theta_\mathrm{s}) - \overline{\mathbf{d}}_\psi(\omega)\right]\left[\mathbf{d}(\omega, \theta_\mathrm{s}) - \overline{\mathbf{d}}_\psi(\omega)\right]^H\right\} \\ &= \mathbf{\Upsilon}_\psi(\omega) - \overline{\mathbf{d}}_\psi(\omega)\overline{\mathbf{d}}_\psi^H(\omega), \end{aligned} \tag{3}$$

where $E[\cdot]$ denotes mathematical expectation, the superscript ^{H} is the conjugate-transpose operator, and

$$\begin{aligned}\mathbf{\Upsilon}_\psi(\omega) &= E\left[\mathbf{d}(\omega, \theta_\mathrm{s})\mathbf{d}^H(\omega, \theta_\mathrm{s})\right] \\ &= \frac{1}{\psi}\int_0^\psi \mathbf{d}(\omega, \theta_\mathrm{s})\mathbf{d}^H(\omega, \theta_\mathrm{s})\, d\theta_\mathrm{s}. \end{aligned} \tag{4}$$

The matrix $\mathbf{\Phi}_{\mathbf{d},\psi}(\omega)$ captures the uncertainties in the random steering vector $\mathbf{d}(\omega, \theta_\mathrm{s})$.

With this signal model, the zero-mean observed signal vector (of length M) is

$$\begin{aligned}\mathbf{y}(\omega) &= \begin{bmatrix} Y_1(\omega) & Y_2(\omega) & \cdots & Y_M(\omega) \end{bmatrix}^T \\ &= \mathbf{x}(\omega) + \mathbf{v}(\omega) \\ &= \mathbf{d}(\omega, \theta_\mathrm{s}) X(\omega) + \mathbf{v}(\omega), \end{aligned} \tag{5}$$

where $Y_m(\omega)$ is the observed signal at the mth microphone, $\mathbf{x}(\omega) = \mathbf{d}(\omega, \theta_\mathrm{s}) X(\omega)$, $X(\omega)$ is the zero-mean desired signal, and $\mathbf{v}(\omega)$ is the zero-mean additive noise signal vector, which is defined analogously to $\mathbf{y}(\omega)$. The desired signal and additive noise are assumed to be uncorrelated with each other. We deduce that the correlation matrix of $\mathbf{y}(\omega)$ is

$$\begin{aligned}\mathbf{\Phi}_\mathbf{y}(\omega) &= E\left[\mathbf{y}(\omega)\mathbf{y}^H(\omega)\right] \\ &= \phi_X(\omega)\mathbf{\Upsilon}_\psi(\omega) + \mathbf{\Phi}_\mathbf{v}(\omega) \\ &= \phi_X(\omega)\mathbf{\Upsilon}_\psi(\omega) + \phi_{V_1}(\omega)\mathbf{\Gamma}_\mathbf{v}(\omega), \end{aligned} \tag{6}$$

where $\phi_X(\omega) = E\left[|X(\omega)|^2\right]$ and $\phi_{V_1}(\omega) = E\left[|V_1(\omega)|^2\right]$ are the variances of $X(\omega)$ and $V_1(\omega)$, respectively, with $V_1(\omega)$ being the noise signal at the first microphone, $\mathbf{\Phi}_\mathbf{v}(\omega) = E\left[\mathbf{v}(\omega)\mathbf{v}^H(\omega)\right]$ is the correlation matrix of $\mathbf{v}(\omega)$, and $\mathbf{\Gamma}_\mathbf{v}(\omega) = \dfrac{\mathbf{\Phi}_\mathbf{v}(\omega)}{\phi_{V_1}(\omega)}$ is the pseudo-coherence matrix of $\mathbf{v}(\omega)$. In the rest, it is assumed that we deal with the diffuse noise, so that

$$\begin{aligned}\mathbf{\Gamma}_\mathbf{v}(\omega) &= \mathbf{\Gamma}_\mathrm{d}(\omega) \\ &= \mathcal{N}_f \int_0^\pi f(\theta)\mathbf{d}(\omega, \theta)\mathbf{d}^H(\omega, \theta)\, d\theta, \end{aligned} \tag{7}$$

where $f(\theta)$ is a positive function on the interval $[0, \pi]$, and

$$\mathcal{N}_f = \frac{1}{\int_0^\pi f(\theta) d\theta} \tag{8}$$

is a normalization parameter. Here, we set $f(\theta) = 1$, which results in the cylindrically isotropic (diffuse) noise field. The (i, j)th element of the $M \times M$ matrix $\mathbf{\Gamma}_\mathrm{d}(\omega)$ in this case is

$$[\mathbf{\Gamma}_\mathrm{d}(\omega)]_{ij} = J_0[\omega(j - i)/c], \tag{9}$$

where $J_0[\cdot]$ is the zero-order Bessel function of the first kind. It should be noted that in many studies about superdirective and differential beamformers [7], [9], [13], the spherically isotropic noise is used to design and evaluate the beamformers. In this work, however, since it is assumed that the microphone array and the desired source are in the same plane and the elevation angle is 0, the cylindrically isotropic noise field is used.

By applying a complex-valued linear filter (of length M), $\mathbf{h}(\omega)$, to the observed signal vector, $\mathbf{y}(\omega)$, we obtain the beamformer output:

$$\begin{aligned}Z(\omega) &= \mathbf{h}^H(\omega)\mathbf{y}(\omega) \\ &= \mathbf{h}^H(\omega)\mathbf{d}(\omega, \theta_\mathrm{s}) X(\omega) + \mathbf{h}^H(\omega)\mathbf{v}(\omega), \end{aligned} \tag{10}$$

where $Z(\omega)$ is the estimate of the desired signal. We deduce that the variance of $Z(\omega)$ is

$$\begin{aligned}&\phi_Z(\omega) \\ &= \mathbf{h}^H(\omega)\mathbf{\Phi}_\mathbf{y}(\omega)\mathbf{h}(\omega) \\ &= \phi_X(\omega)\mathbf{h}^H(\omega)\mathbf{\Upsilon}_\psi(\omega)\mathbf{h}(\omega) + \phi_{V_1}(\omega)\mathbf{h}^H(\omega)\mathbf{\Gamma}_\mathrm{d}(\omega)\mathbf{h}(\omega) \\ &= \phi_X(\omega)\left|\mathbf{h}^H(\omega)\overline{\mathbf{d}}_\psi(\omega)\right|^2 + \phi_X(\omega)\mathbf{h}^H(\omega)\mathbf{\Phi}_{\mathbf{d},\psi}(\omega)\mathbf{h}(\omega) \\ &\quad + \phi_{V_1}(\omega)\mathbf{h}^H(\omega)\mathbf{\Gamma}_\mathrm{d}(\omega)\mathbf{h}(\omega). \end{aligned} \tag{11}$$

It is always possible to exploit the deterministic distortionless constraint:

$$\mathbf{h}^H(\omega)\overline{\mathbf{d}}_\psi(\omega) = 1. \tag{12}$$

III. Performance Measures

The first important measure is the beampattern or directivity pattern, which describes the sensitivity of the beamformer to a plane wave impinging on the array from the direction θ. It is given by

$$\begin{aligned}\mathcal{B}\left[\mathbf{h}(\omega), \theta\right] &= \mathbf{d}^H(\omega, \theta)\mathbf{h}(\omega) \\ &= \sum_{m=1}^M H_m(\omega) e^{\jmath(m-1)\omega\tau_0\cos\theta}, \end{aligned} \tag{13}$$

where $H_m(\omega)$, $m = 1, 2, \ldots, M$ are the complex-valued coefficients of the filter $\mathbf{h}(\omega)$. The lobe with the highest amplitude is named the main lobe. The range between the first nulls on each side of the main lobe is defined as the beamwidth.

If we take Microphone 1 as the reference, we can define the input SNR with respect to this reference as

$$\mathrm{iSNR}(\omega) = \frac{\phi_X(\omega)}{\phi_{V_1}(\omega)}. \tag{14}$$

The output SNR is defined as

$$\mathrm{oSNR}\left[\mathbf{h}(\omega)\right] = \frac{\phi_X(\omega)}{\phi_{V_1}(\omega)} \times \frac{\mathbf{h}^H(\omega)\mathbf{\Upsilon}_\psi(\omega)\mathbf{h}(\omega)}{\mathbf{h}^H(\omega)\mathbf{\Gamma}_\mathrm{d}(\omega)\mathbf{h}(\omega)}. \tag{15}$$

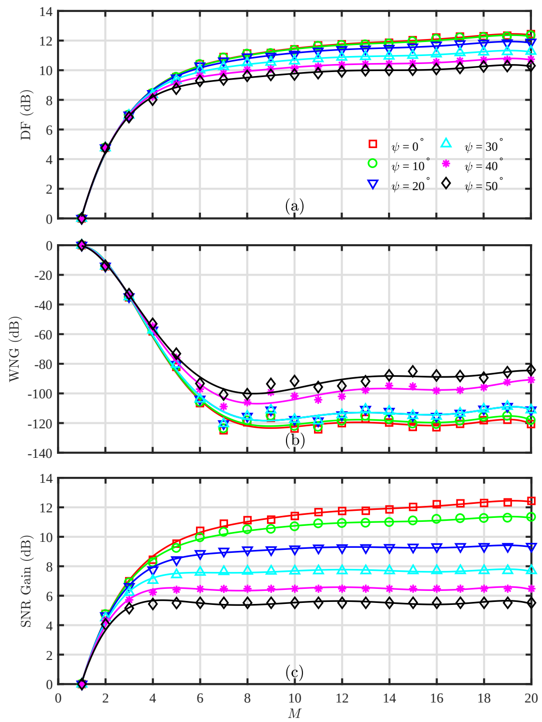

Fig. 1. Performance of the MSNRG beamformer versus M for different values of ψ: (a) DF, (b) WNG, and (c) SNR gain in the cylindrically isotropic noise field. Conditions: $\delta = 1.0$ cm and $f = 1$ kHz.

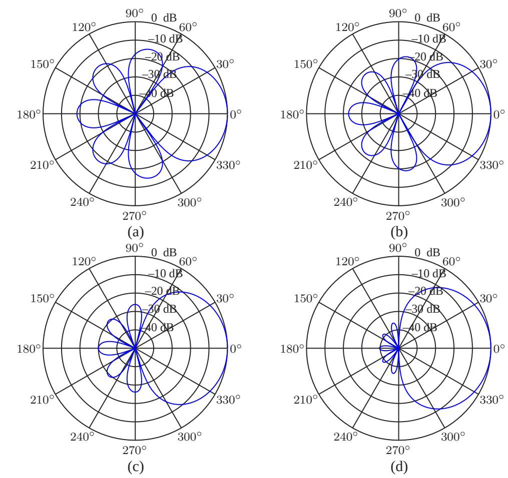

Fig. 2. Beampatterns of the MSNRG beamformer for different values of ψ: (a) $\psi = 20°$, (b) $\psi = 40°$, (c) $\psi = 60°$, and (d) $\psi = 80°$. Conditions: $M = 4$, $\delta = 1.0$ cm, and $f = 1$ kHz.

From the previous definitions, we deduce that the SNR gain is

$$\mathcal{G}[\mathbf{h}(\omega)] = \frac{\text{oSNR}[\mathbf{h}(\omega)]}{\text{iSNR}(\omega)} = \frac{\mathbf{h}^H(\omega)\boldsymbol{\Upsilon}_\psi(\omega)\mathbf{h}(\omega)}{\mathbf{h}^H(\omega)\boldsymbol{\Gamma}_\text{d}(\omega)\mathbf{h}(\omega)} = \frac{\frac{1}{\psi}\int_0^\psi |\mathcal{B}[\mathbf{h}(\omega),\theta]|^2 d\theta}{\frac{1}{\pi}\int_0^\pi |\mathcal{B}[\mathbf{h}(\omega),\theta]|^2 d\theta}, \tag{16}$$

which is closely related to the front-to-back ratio (FBR) [12], [13] defined as

$$\mathcal{R}(\mathbf{h}) = \frac{\int_0^{\pi/2} |\mathcal{B}(\mathbf{h},\theta)|^2 d\theta}{\int_{\pi/2}^{\pi} |\mathcal{B}(\mathbf{h},\theta)|^2 d\theta} \tag{17}$$

in the cylindrically isotropic noise field.

The most convenient way to evaluate the sensitivity of the array to some of its imperfections with conventional superdirective beamformers is via the so-called white noise gain (WNG), which is defined as

$$\mathcal{W}[\mathbf{h}(\omega)] = \frac{\left|\mathbf{h}^H(\omega)\mathbf{d}(\omega,0)\right|^2}{\mathbf{h}^H(\omega)\mathbf{h}(\omega)}, \tag{18}$$

where $\mathbf{d}(\omega,0)$ is the steering vector corresponding to the desired source, which is assumed to be at the endfire direction. The maximum WNG that can be achieved by a beamformer is equal to M [22], i.e., $\mathcal{W}_\text{max}[\mathbf{h}(\omega)] = M$. Note that $\mathcal{W}[\mathbf{h}(\omega)] < 1$ indicates there is white noise amplification.

Another important measure with conventional superdirective beamformers is the DF. It quantifies how the microphone array performs in the presence of reverberation when the desired source is at the endfire direction. With the cylindrically isotropic (diffuse) noise field, the DF is defined as

$$\mathcal{D}[\mathbf{h}(\omega)] = \frac{\left|\mathbf{h}^H(\omega)\mathbf{d}(\omega,0)\right|^2}{\mathbf{h}^H(\omega)\boldsymbol{\Gamma}_\text{d}(\omega)\mathbf{h}(\omega)}. \tag{19}$$

The maximum DF, $\mathcal{D}_\text{max}[\mathbf{h}(\omega)]$, that can be achieved by a beamformer in this case is equal to $2M-1$ [27].

The WNG and DF will also be used here to evaluate the new fixed beamformers.

IV. Superdirective Beamformers

In (16), we recognize the generalized Rayleigh quotient [28]. It is well known that this quotient is maximized with the eigenvector corresponding to the maximum eigenvalue of $\boldsymbol{\Gamma}_\text{d}^{-1}(\omega)\boldsymbol{\Upsilon}_\psi(\omega)$. Therefore, the MSNRG beamformer is

$$\mathbf{h}_\text{MSNRG}(\omega) = \alpha(\omega)\mathbf{t}(\omega), \tag{20}$$

where $\alpha(\omega) \neq 0$ is an arbitrary complex number and $\mathbf{t}(\omega)$ is the eigenvector of matrix $\boldsymbol{\Gamma}_\text{d}^{-1}(\omega)\boldsymbol{\Upsilon}_\psi(\omega)$ corresponding to the maximum eigenvalue, $\lambda(\omega)$. We deduce that

$$\mathcal{G}[\mathbf{h}_\text{MSNRG}(\omega)] = \lambda(\omega). \tag{21}$$

Clearly, we always have

$$\mathcal{G}[\mathbf{h}_\text{MSNRG}(\omega)] \geq \mathcal{G}[\mathbf{h}(\omega)], \ \forall\mathbf{h}(\omega). \tag{22}$$

In practice, it is important to properly choose the value of $\alpha(\omega)$. Here, we have three possibilities.

Let us define the conventional distortionless constraint:

$$\mathbf{h}^H(\omega)\mathbf{d}(\omega,0) = 1. \tag{23}$$

The first possibility is to find $\alpha(\omega)$ in such a way that the MSNRG beamformer is distortionless in the conventional sense. Substituting (20) into (23), we get

$$\alpha(\omega) = \frac{1}{\mathbf{d}^H(\omega,0)\mathbf{t}(\omega)}. \tag{24}$$

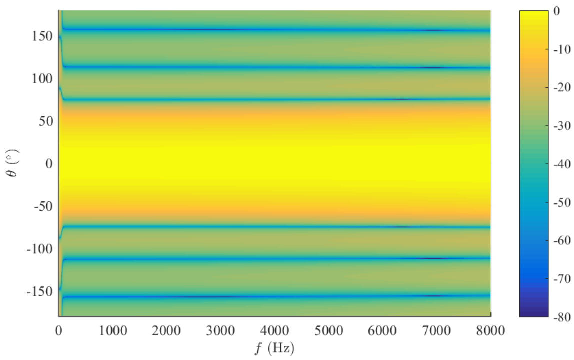

Fig. 3. Beampatterns of the MSNRG beamformer versus frequency. Conditions: $M = 4$, $\delta = 1.0$ cm, and $\psi = 60°$.

Plugging (24) in (20), we obtain the first kind of MSNRG beamformer:

$$\mathbf{h}_{\text{MSNRG},1}(\omega) = \frac{\mathbf{t}(\omega)}{\mathbf{d}^H(\omega, 0)\,\mathbf{t}(\omega)}. \tag{25}$$

The second possibility to find $\alpha(\omega)$ is by using the deterministic distortionless constraint defined in (12). We easily find the second MSNRG beamformer:

$$\mathbf{h}_{\text{MSNRG},2}(\omega) = \frac{\mathbf{t}(\omega)}{\overline{\mathbf{d}}_{\psi}^H(\omega)\,\mathbf{t}(\omega)}. \tag{26}$$

Finally, the third and last proposed idea to find $\alpha(\omega)$ is by using the power conservation constraint:

$$\mathbf{h}^H(\omega)\,\mathbf{\Upsilon}_{\psi}(\omega)\,\mathbf{h}(\omega) = 1. \tag{27}$$

Then, the third MSNRG beamformer is

$$\mathbf{h}_{\text{MSNRG},3}(\omega) = \frac{\mathbf{t}(\omega)}{\sqrt{\mathbf{t}^H(\omega)\,\mathbf{\Upsilon}_{\psi}(\omega)\,\mathbf{t}(\omega)}}. \tag{28}$$

It should be noted that, when $\psi = 0$, i.e., the fixed point source case, the proposed three MSNRG beamformers degenerate to the traditional SD beamformer [7], [22].

V. Simulations

In this section, the performance of the proposed beamformers are evaluated in terms of the beampattern, SNR gain, WNG, and DF.

First, we examine the DF, WNG, and SNR gain of the MSNRG beamformer versus the number of microphones, M, for $f = 1$ kHz and different values of ψ. A uniform linear array (ULA) with $\delta = 1.0$ cm is considered. The performances of the proposed three MSNRG beamformers are very similar, which is reasonable since the only difference among them is the choice of the scalar $\alpha(\omega)$. So, we just present the performance of the first kind of MSNRG beamformers, $\mathbf{h}_{\text{MSNRG},1}(\omega)$. As can be seen, when $\psi > 0$, the DF of the beamformer $\mathbf{h}_{\text{MSNRG},1}(\omega)$ is smaller than that of the traditional SD beamformer ($\psi = 0$), and the difference increases with the value of ψ and the number of microphones. Specifically, when ψ is large, the SNR gain of the MSNRG beamformer in the cylindrically isotropic noise field does not increase with M after reaching its maximum, and so is the WNG. For example, when $\psi = 50°$, the maximum SNR gain in the cylindrically isotropic noise field is obtained when $M = 6$ and this gain does no longer increase with the number of microphones. The WNG stays in a very low level no matter how many microphones are used. Both the DF and SNR gain decrease with the increase of ψ, while the WNG increases with the increase of ψ.

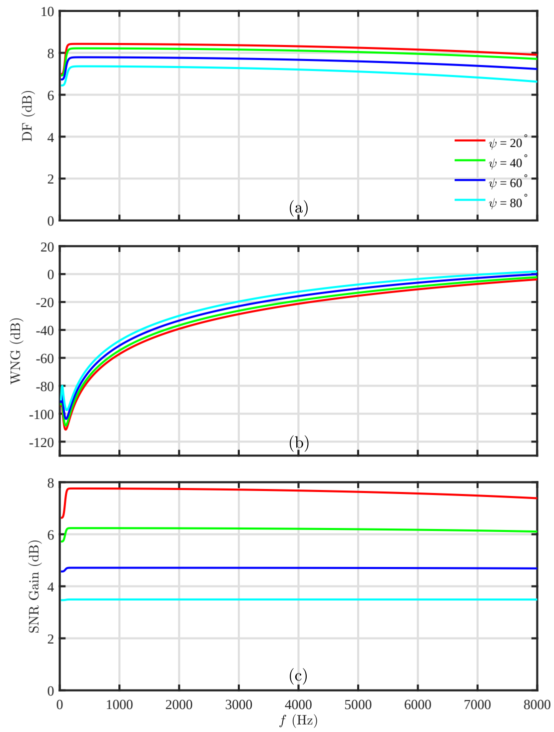

Fig. 4. Performance of the MSNRG beamformer versus frequency for different values of ψ: (a) DF, (b) WNG, and (c) SNR gain in the cylindrically isotropic noise field. Conditions: $M = 4$ and $\delta = 1.0$ cm

Second, we consider a ULA with $M = 4$ and $\delta = 1.0$ cm. The beampatterns of the beamformer $\mathbf{h}_{\text{MSNRG},1}(\omega)$ for $f = 1$ kHz and different values of ψ are plotted in Fig. 2. As seen, the beampatterns of the MSNRG beamformer vary greatly with ψ. The beamwidth of the beampattern increases with the value of ψ. Accordingly, the beamwidth of the designed beampattern can be changed by controlling the value of ψ. It should be noted, however, that for the given array aperture, the beamwidth of the designed beampattern cannot be very narrow, even when $\psi = 0$, due to the small aperture of the array. Figure 3 plots the beampatterns of the designed MSNRG beamformer versus frequency for $\psi = 60°$. As seen, the obtained beampatterns are almost frequency invariant, except at very low frequencies ($f < 100$ Hz) due to numerical problems. The simulation results, i.e., DF, WNG, and SNR gain of the MSNRG beamformer, $\mathbf{h}_{\text{MSNRG},1}(\omega)$, in the cylindrically isotropic noise field versus frequency with different values of ψ are plotted in Fig. 4. It shows that the DF decreases with the value of ψ. This is reasonable since the wider is the beamwidth, the lower is the DF. Also, the SNR gain in the cylindrically isotropic noise field decreases with the value of ψ, while the WNG increases with the increase of the value of ψ. From Figs 1 and 4, we can see that, even though the MSNRG beamformer can achieve the maximum SNR gain and high DF for different values of ψ, it suffers from serious white noise amplification (very low WNG), even when we use more number of microphones. Our work in progress is to investigate how to improve the WNG, while maintain the SNR gain in the cylindrically isotropic noise field and frequency-invariant beampattern of the MSNRG beamformer by exploiting the redundancy provided by more number of microphones.

VI. Summary

We have derived three MSNRG beamformers under some uncertainties in the DoA of the desired source. According to the simulations, we cannot always improve the SNR gain of the MSNRG beamformer by increasing the number of microphones when ψ is large, which is in contrast to the fixed point source case. The beamwidth of the MSNRG beamformer can cover the distributed range of the desired source. The MSNRG beamformer achieves the maximum SNR gain and almost frequency-invariant beampatterns. The traditional SD beamformer is a particular case of the proposed framework. While we consider only linear arrays, the principle of the presented beamformers can be generalized to circular and spherical geometries straightforwardly.

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