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Gongping Huang
Technion - Israel Institute of Technology. הטכניון

Jingdong Chen
Institute of Electrical and Electronics Engineers

Jacob Benesty
Institut National de la Recherche Scientifique

ON THE DESIGN OF ROBUST STEERABLE FREQUENCY-INVARIANT BEAMPATTERNS WITH CONCENTRIC CIRCULAR MICROPHONE ARRAYS

Gongping Huang[1], *Jingdong Chen*[1], *and Jacob Benesty*[2]

[1]CIAIC
Northwestern Polytechnical University
Xi'an, Shaanxi 710072, China

[2]INRS-EMT, University of Quebec
800 de la Gauchetiere Ouest, Suite 6900
Montreal, QC H5A 1K6, Canada

ABSTRACT

This paper studies the problem of frequency-invariant beamforming with concentric circular microphone arrays (CCMAs). We develop a beamforming algorithm based on an optimal approximation of the beamformer's beampattern with the Jacobi-Anger expansion. In comparison with the existing frequency-invariant beamformers with either circular microphone arrays (CMAs) or CCMAs, the developed algorithm offers the following advantages: 1) it can mitigate the deep-null problem encountered in CMAs and therefore has a consistent directivity factor over the frequency range of speech signals; 2) it is more flexible in terms of steering flexibility and the resulting beampattern can be steered to any direction; and 3) it does not require the microphones in different rings of the CCMA to be aligned, which is very useful in practice, particularly when microphone arrays with small and compact apertures have to be used.

Index Terms— Microphone arrays, concentric circular microphone arrays, fixed beamforming, frequency-invariant beampattern, white noise gain, directivity factor.

1. INTRODUCTION

Microphone array beamforming, an important problem in acoustic signal processing for voice communications and human-machine interfaces, has attracted a considerable amount of attention over the past few decades. Many beamforming algorithms have been developed [1–14]. In real applications, circular microphone arrays (CMAs) are often used due to their steering ability [15–20]. One type of CMAs, i.e., circular differential microphone arrays (CDMAs), have been shown to be particularly useful in speech and audio applications since they can form frequency-invariant beampatterns and attain high directional gains [21–26]. However, CDMAs may suffer from the so-called deep-null problem, which is more serious at high frequencies. This has become a limiting factor that restricts the application of CDMAs in practical systems. Recently, a robust beamforming algorithm was developed with the use of concentric CDMAs (CCDMAs). It can deal with the deep-null problem while achieving good performance over the frequency range of interest. But the resulting beampattern can only be steered to a limited number of directions [26]. Moreover, with the existing methods, the geometry of the CCDMA must satisfy perfect symmetry, i.e., 1) the number of microphones in outer rings must be integral multiples of the number of microphones in inner rings; and 2) microphones in different rings need to be aligned. To deal with the aforementioned limitations, we develop a frequency-invariant beamforming algorithm with CCMAs based on the Jacobi-Anger series expansions [17]. This method offers the following advantages in comparison with existing methods: 1) it has full steering flexibility, i.e., the directivity pattern can be steered to any directions in the plane in which the sensors are located; 2) it can mitigate the deep-null problem; and 3) it does not require alignment of microphones in different rings, which is very useful in practical applications, especially when CCMAs with small and compact apertures are used.

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2. SIGNAL MODEL, PROBLEM FORMULATION, AND PERFORMANCE MEASURES

Considering a farfield source signal (plane wave), that propagates in an anechoic acoustic environment at the speed of sound, i.e., $c = 340$ m/s, and impinges on a CCMA composed of P rings, where the pth ($p = 1, 2, \ldots, P$) ring, with a radius of r_p, consists of M_p omnidirectional microphones. Without loss of generality, we assume that all the sensors are in the horizonal plane, the center of the CCMA coincides with the origin of the Cartesian coordinate system, azimuthal angles are measured anticlockwise from the positive direction of x axis, and sensor 1 of the array is placed on the positive side of the x axis. The direction of the source signal to the array is parameterized by the azimuth angle, θ. In this scenario, the steering vector of length $\underline{M}$, where $\underline{M} = \sum_{p=1}^{P} M_p$ is the total number of microphones, is defined as [1, 26]

$$\underline{\mathbf{d}}(\omega,\theta) = \left[\, \mathbf{d}_1^T(\omega,\theta) \;\; \mathbf{d}_2^T(\omega,\theta) \;\; \cdots \;\; \mathbf{d}_P^T(\omega,\theta) \, \right]^T, \qquad (1)$$

where the superscript ^{T} is the transpose operator,

$$\mathbf{d}_p(\omega,\theta) = \Big[e^{\jmath \varpi_p \cos(\theta - \psi_{p,1})} \;\; e^{\jmath \varpi_p \cos(\theta - \psi_{p,2})} \;\; \cdots \;\; e^{\jmath \varpi_p \cos\left(\theta - \psi_{p,M_p}\right)} \Big]^T \qquad (2)$$

is the pth ring's steering vector, $\jmath$ is the imaginary unit with $\jmath^2 = -1$, $\varpi_p = \omega r_p / c$, with $\omega = 2\pi f$ being the angular frequency and $f > 0$ being the temporal frequency, and

$$\psi_{p,m} = \frac{2\pi(m-1)}{M_p} \qquad (3)$$

is the angular position of the mth ($m = 1, 2, \ldots, M_p$) array element on the pth ($p = 1, 2, \ldots, P$) ring. In order to avoid spatial aliasing, it is necessary that the interelement spacing is less than half of the acoustic wavelength. In this paper, we consider fixed beamformers with small values of the interelement spacing, so that this condition easily holds [3, 5].

Beamforming aims at recovering a signal of interest (also called the desired signal) from the noisy observation vector. In this paper, we consider the general case where the desired signal comes from the direction $\theta_{\rm s}$ and the corresponding propagation vector is $\underline{\mathbf{d}}\left(\omega, \theta_{\rm s}\right)$. Then, our objective is to design a desired, frequency-invariant beampattern with its main beam pointing to the direction $\theta_{\rm s}$. To do that, a complex weight, $H_{p,m}^{*}(\omega)$, is applied to the output of the mth sensor on the pth ring, where the superscript * denotes complex conjugation. The weighted outputs are then summed together to form the beamformer's output. The weights can be put together into a vector of length $\underline{M}$ as

$$\underline{\mathbf{h}}\left(\omega\right) = \left[\begin{array}{cccc} \mathbf{h}_1^T\left(\omega\right) & \mathbf{h}_2^T\left(\omega\right) & \cdots & \mathbf{h}_P^T\left(\omega\right) \end{array}\right]^T, \quad (4)$$

where

$$\mathbf{h}_p\left(\omega\right) = \left[\begin{array}{cccc} H_{p,1}(\omega) & H_{p,2}(\omega) & \cdots & H_{p,M_p}(\omega) \end{array}\right]^T \quad (5)$$

is the weighting vector on the pth ring.

To let the source signal pass through the beamformer without distortion, the distortionless constraint in the desired direction is needed, i.e.,

$$\underline{\mathbf{h}}^H\left(\omega\right)\underline{\mathbf{d}}\left(\omega, \theta_{\rm s}\right) = 1, \quad (6)$$

where the superscript ^{H} is the conjugate-transpose operator.

Now, we give some useful measures, i.e., the beampattern (or directivity pattern), the directivity factor (DF), and the white noise gain (WNG), to evaluate the performance of the proposed beamformer.

The beampattern describes the sensitivity of the fixed beamformer to a plane wave impinging on the CCMA from the direction θ [5]. Mathematically, it is defined as

$$\begin{aligned} \mathcal{B}\left[\underline{\mathbf{h}}\left(\omega\right), \theta\right] &= \underline{\mathbf{h}}^H\left(\omega\right)\underline{\mathbf{d}}\left(\omega, \theta\right) \quad (7)\\ &= \sum_{p=1}^{P}\sum_{m=1}^{M_p} H_{p,m}^{*}\left(\omega\right) e^{\jmath \varpi_p \cos\left(\theta - \psi_{p,m}\right)}. \end{aligned}$$

The DF quantifies the ability of the beamformer in suppressing spatial noise from directions other than the look direction. It is written as [3, 5]

$$\mathcal{D}\left[\underline{\mathbf{h}}\left(\omega\right)\right] = \frac{\left|\underline{\mathbf{h}}^H\left(\omega\right)\underline{\mathbf{d}}\left(\omega, \theta_{\rm s}\right)\right|^2}{\underline{\mathbf{h}}^H\left(\omega\right)\mathbf{\Gamma}_{\rm d}\left(\omega\right)\underline{\mathbf{h}}\left(\omega\right)}, \quad (8)$$

where $\mathbf{\Gamma}_{\rm d}\left(\omega\right)$ is the pseudo-coherence matrix of the noise signal in a diffuse (spherically isotropic) noise field, and the (i, j)th element of $\mathbf{\Gamma}_{\rm d}\left(\omega\right)$ is

$$\left[\mathbf{\Gamma}_{\rm d}\left(\omega\right)\right]_{ij} = \mathrm{sinc}\left(\frac{\omega\delta_{ij}}{c}\right), \quad (9)$$

with $\delta_{ij} = \left\|\mathbf{r}_i - \mathbf{r}_j\right\|_2$ being the distance between microphone i and j, $\|\cdot\|_2$ being the Euclidean norm, $\mathbf{r}_i, \mathbf{r}_j \in \{\mathbf{r}_{1,1}, \mathbf{r}_{1,2}, \ldots, \mathbf{r}_{p,M_p}, \ldots, \mathbf{r}_{P,M_P}\}$, and $\mathbf{r}_{p,m}$ is the coordinates of the mth microphone at the pth ring.

The WNG evaluates the sensitivity of a beamformer to some of its imperfections. It can be written as [3]

$$\mathcal{W}\left[\underline{\mathbf{h}}\left(\omega\right)\right] = \frac{\left|\underline{\mathbf{h}}^H\left(\omega\right)\underline{\mathbf{d}}\left(\omega, \theta_{\rm s}\right)\right|^2}{\underline{\mathbf{h}}^H\left(\omega\right)\underline{\mathbf{h}}\left(\omega\right)}. \quad (10)$$

3. DESIRED BEAMPATTERNS

In our context, the objective of beamforming is to find a proper beamforming filter, $\underline{\mathbf{h}}\left(\omega\right)$, so that its beampattern is as close as possible to a desired (target) frequency-invariant beampattern. In acoustic and speech applications, we generally use the frequency-invariant beampatterns of different orders that were developed in the context of differential microphone arrays (DMAs) as the desired beampatterns [2, 4, 5]. As shown in [5], an Nth-order frequency-invariant beampattern with its main beam pointing to the direction of 0° is given by

$$\mathcal{B}\left(\mathbf{a}_N, \theta\right) = \sum_{n=0}^{N} a_{N,n}\cos\left(n\theta\right) = \mathbf{a}_N^T\mathbf{p}_{\rm c}\left(\theta\right), \quad (11)$$

where $a_{N,n},\ n = 0, 1, \ldots, N$, are real coefficients, and

$$\begin{aligned} \mathbf{a}_N &= \left[\begin{array}{cccc} a_{N,0} & a_{N,1} & \cdots & a_{N,N} \end{array}\right]^T, \\ \mathbf{p}_{\rm c}\left(\theta\right) &= \left[\begin{array}{cccc} 1 & \cos\theta & \cdots & \cos\left(N\theta\right) \end{array}\right]^T. \end{aligned}$$

It can be checked that $\mathcal{B}\left(\mathbf{a}_N, \theta\right)$ is symmetric about the axis $0 \leftrightarrow \pi$. The values of the coefficients $a_{N,n},\ n = 0, 1, \ldots, N$, in (11) determine the shape of the directivity pattern as well as the corresponding DF. In the direction of the main beam, i.e., $\theta = 0^\circ$, the directivity pattern should be equal to 1, i.e., $\mathcal{B}\left(\mathbf{a}_N, 0^\circ\right) = 1$. Therefore, we have

$$\sum_{n=0}^{N} a_{N,n} = 1. \quad (12)$$

We write the directivity pattern corresponding to a steering angle $\theta_{\rm s}$ as [17]:

$$\begin{aligned} \mathcal{B}\left(\mathbf{b}_{2N}, \theta - \theta_{\rm s}\right) &= \sum_{n=-N}^{N} b_{2N,n} e^{\jmath n\left(\theta - \theta_{\rm s}\right)} \quad (13)\\ &= \left[\mathbf{\Upsilon}\left(\theta_{\rm s}\right)\mathbf{b}_{2N}\right]^T\mathbf{p}_e\left(\theta\right) \\ &= \mathbf{c}_{2N}^T\left(\theta_{\rm s}\right)\mathbf{p}_e\left(\theta\right) \\ &= \mathcal{B}\left[\mathbf{c}_{2N}\left(\theta_{\rm s}\right), \theta\right], \end{aligned}$$

where

$$\left\{\begin{array}{l} b_{2N,0} = a_{N,0} \\ b_{2N,i} = b_{2N,-i} = \dfrac{1}{2}a_{N,i},\ i = 1, 2, \ldots, N \end{array}\right.,$$

and

$$\begin{aligned} \mathbf{\Upsilon}\left(\theta_{\rm s}\right) &= \mathrm{diag}\left(e^{\jmath N\theta_{\rm s}}, \ldots, 1, \ldots, e^{-\jmath N\theta_{\rm s}}\right), \\ \mathbf{b}_{2N} &= \left[\begin{array}{ccccc} b_{2N,-N} & \cdots & b_{2N,0} & \cdots & b_{2N,N} \end{array}\right]^T, \\ \mathbf{p}_e\left(\theta\right) &= \left[\begin{array}{ccccc} e^{-\jmath N\theta} & \cdots & 1 & \cdots & e^{\jmath N\theta} \end{array}\right]^T, \\ \mathbf{c}_{2N}\left(\theta_{\rm s}\right) &= \mathbf{\Upsilon}\left(\theta_{\rm s}\right)\mathbf{b}_{2N} \\ &= \left[c_{2N,-N}\left(\theta_{\rm s}\right) \ \cdots \ c_{2N,0}\left(\theta_{\rm s}\right) \ \cdots \ c_{2N,N}\left(\theta_{\rm s}\right)\right]^T. \end{aligned}$$

Clearly, the main beam of (13) points in the direction $\theta_{\rm s}$ and $\mathcal{B}\left(\mathbf{b}_{2N}, \theta - \theta_{\rm s}\right)$ is symmetric about the axis $\theta_{\rm s} \leftrightarrow \theta_{\rm s} + \pi$. From (13), it is clearly seen that a rotation of the directivity pattern corresponds to a simple modification of its coefficients.

4. BEAMPATTERN DESIGN

In this section, we show how to derive the beamforming filter, $\underline{\mathbf{h}}(\omega)$, such that the designed beampattern, $\mathcal{B}[\underline{\mathbf{h}}(\omega), \theta]$, approaches the desired symmetric directivity pattern, $\mathcal{B}(\mathbf{a}_N, \theta)$.

It has been shown that the optimal approximation of the beamformer's beampattern with a CMA from a least-squares error (LSE) perspective is the Jacobi-Anger expansion [17]. Following this principle, the Jacobi-Anger expansions of the exponential function in the beamformer's beampattern with a CCMA is [27]

$$e^{\jmath \varpi_p \cos(\theta - \psi_{p,m})} = \sum_{n=-\infty}^{\infty} \jmath^n J_n(\varpi_p) e^{\jmath n (\theta - \psi_{p,m})}, \quad (14)$$

where $J_n(\varpi_p)$ is the nth-order Bessel function of the first kind with $J_{-n}(\varpi_p) = (-1)^n J_n(\varpi_p)$. Substituting (14) into the definition of the beamformer's beampattern with a CCMA in (7) and limiting the Jacobi-Anger series to the order N, we obtain

$$\begin{aligned} \mathcal{B}_N[\underline{\mathbf{h}}(\omega), \theta] &\approx \sum_{p=1}^{P} \sum_{m=1}^{M_p} H_{p,m}^*(\omega) \sum_{n=-N}^{N} \jmath^n J_n(\varpi_p) e^{\jmath n (\theta - \psi_{p,m})} \\ &= \sum_{n=-N}^{N} e^{\jmath n \theta} \jmath^n \sum_{p=1}^{P} \sum_{m=1}^{M_p} J_n(\varpi_p) e^{-\jmath n \psi_{p,m}} H_{p,m}^*(\omega) \\ &= \sum_{n=-N}^{N} e^{\jmath n \theta} c_{2N,n}(\theta_{\mathrm{s}}), \quad (15) \end{aligned}$$

where

$$\jmath^n \sum_{p=1}^{P} \sum_{m=1}^{M_p} J_n(\varpi_p) e^{-\jmath n \psi_{p,m}} H_{p,m}^*(\omega) = c_{2N,n}(\theta_{\mathrm{s}}). \quad (16)$$

This gives the relation between the beamformer's beampattern with a CCMA and the Nth order desired directivity pattern. Based upon this relationship, the beamforming filter, $\underline{\mathbf{h}}(\omega)$, can be obtained through solving a linear equation constructed from the optimal approximation of the beampattern with the Jacobi-Anger series expansions. By writing (16) in a vector form, we have

$$\jmath^n \underline{\boldsymbol{\psi}}_n^T(\omega) \underline{\mathbf{h}}^*(\omega) \quad = c_{2N,n}(\theta_{\mathrm{s}}), \quad (17)$$

where

$$\underline{\boldsymbol{\psi}}_n(\omega) = \Big[J_n(\varpi_1) \boldsymbol{\psi}_{n,1}^T, \;\; J_n(\varpi_2) \boldsymbol{\psi}_{n,2}^T \\ \ldots, J_n(\varpi_P) \boldsymbol{\psi}_{n,P}^T \Big]^T \quad (18)$$

is a vector of length $\underline{M}$, with $n = \pm 1, \pm 2, \ldots, \pm N$,

$$\boldsymbol{\psi}_{n,p} = \left[\begin{array}{cccc} e^{-\jmath n \psi_{p,1}} & e^{-\jmath n \psi_{p,2}} & \cdots & e^{-\jmath n \psi_{p,M_p}} \end{array} \right]^T, \quad (19)$$

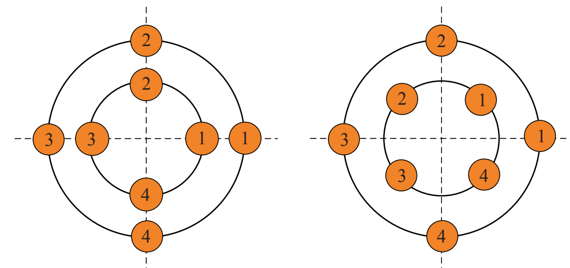
and $p = 1, 2, \ldots, P$.

From (17), we obtain the following equation:

$$\underline{\boldsymbol{\Psi}}(\omega) \underline{\mathbf{h}}(\omega) = \mathbf{J}^* \boldsymbol{\Upsilon}^*(\theta_{\mathrm{s}}) \mathbf{b}_{2N}, \quad (20)$$

where

$$\mathbf{J} = \mathrm{diag}\left[\frac{1}{\jmath^{-N}}, \ldots, 1, \ldots, \frac{1}{\jmath^N} \right] \quad (21)$$



(a) (b)

Fig. 1. Illustration of a CCMA with two rings, where each ring consists of 4 omnidirectional microphones: (a) microphones in two rings are aligned and (b) microphones in two rings are not aligned.

is a $(2N+1) \times (2N+1)$ diagonal matrix and

$$\underline{\boldsymbol{\Psi}}(\omega) = \begin{bmatrix} \underline{\boldsymbol{\psi}}_{-N}^H(\omega) \\ \vdots \\ \underline{\boldsymbol{\psi}}_0^H(\omega) \\ \vdots \\ \underline{\boldsymbol{\psi}}_N^H(\omega) \end{bmatrix} \quad (22)$$

is a $(2N+1) \times \underline{M}$ matrix.

Generally, with a CCMA, we assume that $P \geq 2$ and $\underline{M} > (2N+1)$. The minimum-norm solution of (20) is then

$$\underline{\mathbf{h}}_{\mathrm{MN}}(\omega) = \underline{\boldsymbol{\Psi}}^H(\omega) \left[\underline{\boldsymbol{\Psi}}(\omega) \underline{\boldsymbol{\Psi}}^H(\omega) \right]^{-1} \mathbf{J}^* \boldsymbol{\Upsilon}^*(\theta_{\mathrm{s}}) \mathbf{b}_{2N}. \quad (23)$$

With (23), we can design any desired symmetric directivity pattern with a CCMA. For simplicity, in the reminder of the paper, we call the proposed beamformer as FIB-CCMA.

In comparison with the conventional frequency-invariant beamformer, i.e., the CCDMA beamformers [26], the proposed beamformer have the following advantages.

1) Full steering flexibility. The proposed FIB-CCMA can perfectly steer the beampattern to any look direction in the sensor plane. In comparison, the beamformer in [26] can perfectly steer only to M_1 different directions, i.e., $2\pi(m-1)/M_1$.
2) Structural flexibility. The algorithms in [26] requires the microphones in different rings of the CCDMA to be aligned as shown illustrated in Fig. 1(a). The proposed FIB-CCMA does not need this requirement as illustrated in Fig. 1(b). We can set the angular position of the mth array element on the pth ring [in (3)] as

$$\psi_{p,m} = \psi_{p,0} + \frac{2\pi(m-1)}{M_p}. \quad (24)$$

The fact that the positions of microphones in different rings do not need to be aligned gives much flexibility in designing an array in practical applications. This is very useful especially when using CCMAs with small and compact apertures.

5. SIMULATIONS

In this section, we study the performance of the developed beamforming algorithm for the design of the first-order hypercardioid [5].

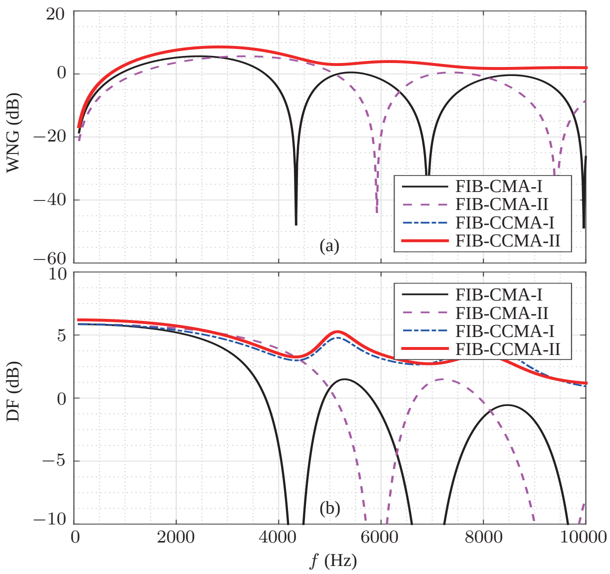


Fig. 2. DF and WNG of the FIB-CMA and FIB-CCMA for the design of the first-order hypercardioid: (a) DF and (b) WNG. Conditions of simulation: $\theta_{\rm s} = 0^\circ$.

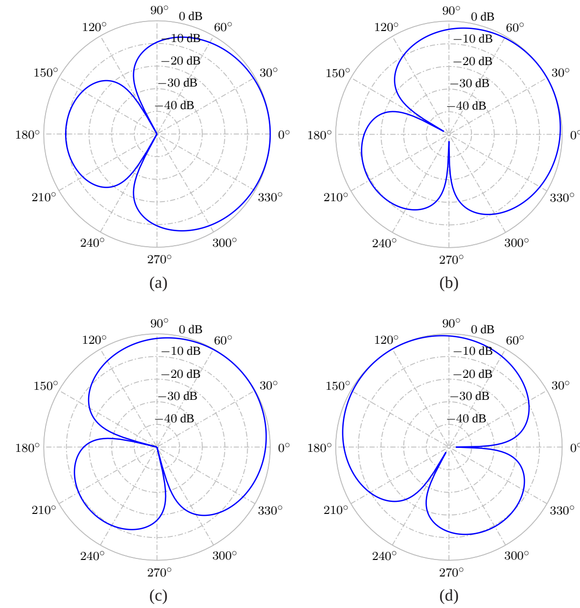


Fig. 3. Beampatterns of the FIB-CCMA for different steering directions, $\theta_{\rm s}$: (a) $\theta_{\rm s} = 0^\circ$, (b) $\theta_{\rm s} = 30^\circ$, (c) $\theta_{\rm s} = 45^\circ$, and (d) $\theta_{\rm s} = 120^\circ$. Conditions of simulation: $f = 1000$ Hz.

The desired beampattern is given by (13) where $N = 1$ and the coefficients are given by $\mathbf{b}_{2N} = [1/3 \;\; 1/3 \;\; 1/3]^T$. We compare the performance of the FIB-CCMA with the FIB-CMA (a special case of the FIB-CCMAs with one ring). The two FIB-CMAs are designed with $M = 4$, $r = 3.0$ cm (FIB-CMA-I), and $M = 4$ and $r = 2.2$ cm (FIB-CMA-II), respectively. The FIB-CCMA is configured with two rings, which is a combination of the two CMAs, i.e., $M_1 = 4$, $M_2 = 4$, $r_1 = 3.0$ cm, $r_2 = 2.2$ cm. For comparison, we consider two cases: 1) two rings are aligned [as showed in Fig. 1 (a)], i.e., $\psi_{1,0} = \psi_{2,0} = 0$ (FIB-CCMA-I); 2) two rings are not aligned [as showed in Fig. 1 (b)], i.e., $\psi_{1,0} = 0$, $\psi_{2,0} = 45^\circ$ (FIB-CCMA-II).

Figure 2 gives plots of the DFs and WNGs of the FIB-CMAs and FIB-CCMAs. As one can see, both FIB-CMAs suffers from serious degradation in DFs and WNGs due to the so-called nulls problem [17, 26]. This is because the denominators of the filter coefficients are a function of Bessel functions, and the zeros of the Bessel function leads to nulls [17, 26]. This problem is more serious with FIB-CMA-I than with FIB-CMA-II because the increase of the array aperture (radius) leads to more nulls in the frequency range of interest [26]. In comparison, the two FIB-CCMAs have almost frequency-invariant performance in the studied frequency range, which verifies that the use of CCMAs can help mitigate the deep nulls problem. The two FIB-CCMAs have similar performance.

Figure 3 plots beampatterns of FIB-CCMA-I with $\theta_{\rm s} \in \{0^\circ, 30^\circ, 45^\circ, 120^\circ\}$. As can be seen, the FIB-CCMA-I has identical beampattern in different directions and all beampatterns are symmetric about the axis $\theta_{\rm s} \leftrightarrow \theta_{\rm s} + \pi$.

6. CONCLUSIONS

In this paper, we studied the problem of designing frequency-invariant beamformers with CCMAs. We proposed an FIB-CCMA algorithm based on an optimal approximation of the beampattern with the Jacobi-Anger series expansions. The developed beamformer can mitigate the deep nulls problem as compared to to existing methods with CDMAs and the deduced beampattern can be steered to any wanted direction in the plane where the sensors are located. Moreover, the developed method does not require to align the microphones in different rings, which is convenient and useful in real applications, particularly when arrays with small and compact apertures have to be used. Simulation results demonstrated the advantage of this proposed algorithm over conventional frequency-invariant beamformers with either CMAs or CCMAs.

7. RELATION TO PRIOR WORK

Microphone array beamforming has long been a very important problem in acoustic, speech, and audio signal processing and many beamforming algorithms have been developed over the last few decades, such as the delay-and-sum, filter-and-sum, superdirective, and differential beamformers [10, 12, 31–33]. Among those, beamformers with CDMAs have attracted much R&D interest for their frequency-invariant response and steering flexibility [21, 23–25]. However, conventional beamformers with CDMAs, e.g., algorithms in [21], suffers from two problems: 1) the deep-null problem in both DF and WNG (so the beamformer cannot perform consistently and robustly at different frequencies), and 2) limited steering ability (so the beampattern, without any changes, can only be steered to a limited number of directions). To deal with the first problem, we studied in [17] beamforming with CCMAs. The resulting beamformers are free from deep nulls in either DF or WNG but they lack steering flexibility. To deal with the second problem, we recently developed a new approach with CDMAs in [26]. The resulting beamformer has full steering flexibility, i.e., the beampattern can be perfectly steered to any directions, but it still suffers from the deep-null problem. This paper is basically a generalization of the work in [17] and [26]. We developed an FIB-CCMA algorithm based on Jacobi-Anger series expansions, which on the one hand does no longer suffer from the deep-null problem, and on the other hand has full steering flexibility. Moreover, it does not require alignment of microphones in different rings of the CCMA.

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